Basic Interpolation and Geostatistics

Interpolation in GeoScene3D

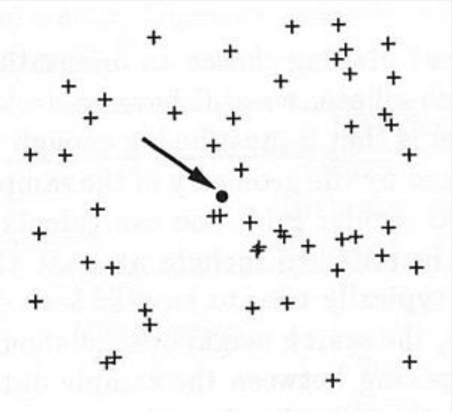
Agenda

- Interpolation defined
- General workflow in GeoScene3D
- Interpolation methods: Nearest-Neighbour (NN)
- Interpolation methods: Inverse Distance Weighted (IDW)
- Interpolation methods: Kriging
- Summing up

GeoScene3D - Interpolation

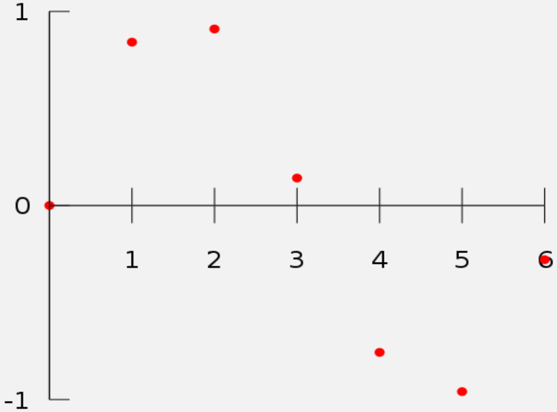
Interpolation

- A method of constructing new data points within the range of a discrete set of known data points.
- Interpolation is an estimation of values within known values from a sequence of values (surrounding known values)
- Irregular datapoints are interpolated to a regular grid



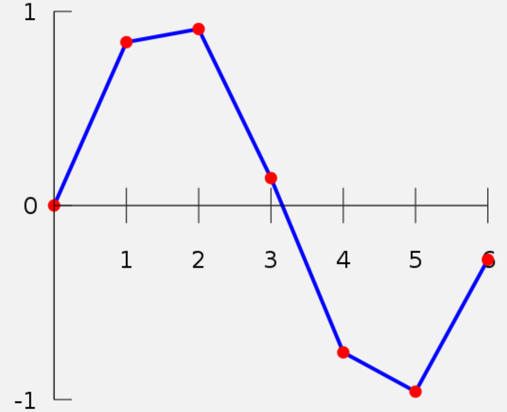
Known Data

x	*f*(*x*)
0	0
1	0.8415
2	0.9093
3	0.1411
4	−0.7568
5	−0.9589
6	−0.2794



Interpolation Method

Interpolated Estimate

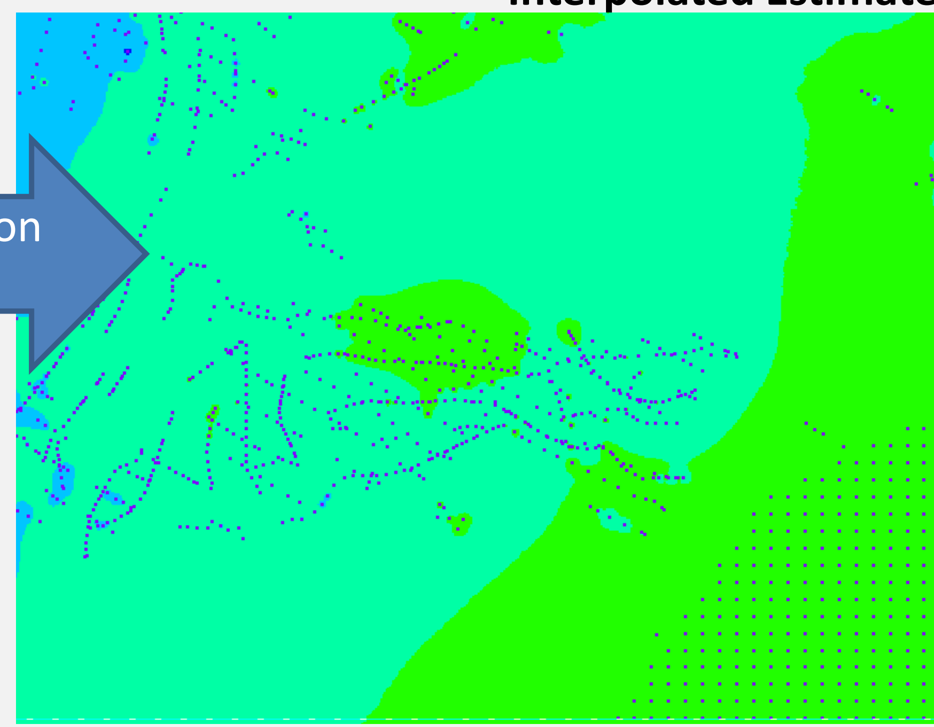


GeoScene3D - Interpolation

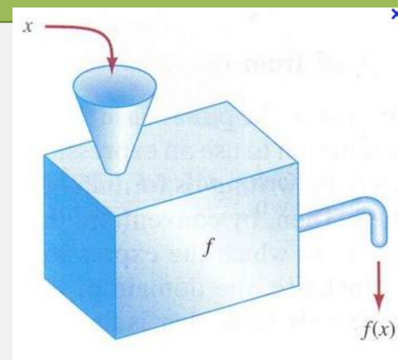
Known Data

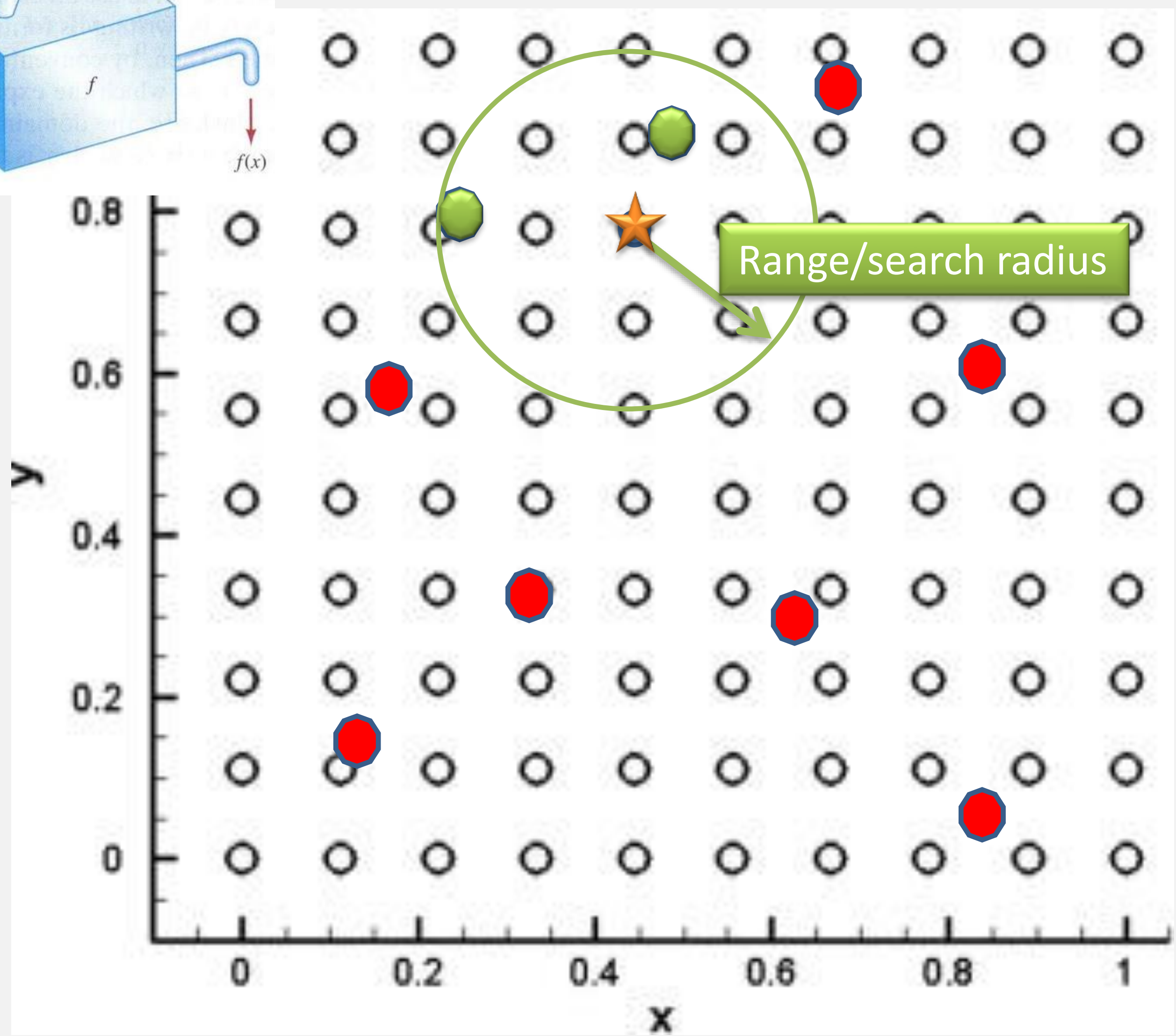
Interpolation Method

Interpolated Estimate



GeoScene3D - Interpolation





We have our *data*

We have our empty *grid*

We estimate values in each *node*

Using an *interpolation method*

Whithin have a *search range*

Which we use to *define* the data to use in the interpolation.

And *calculate an estimate* In the cell node!

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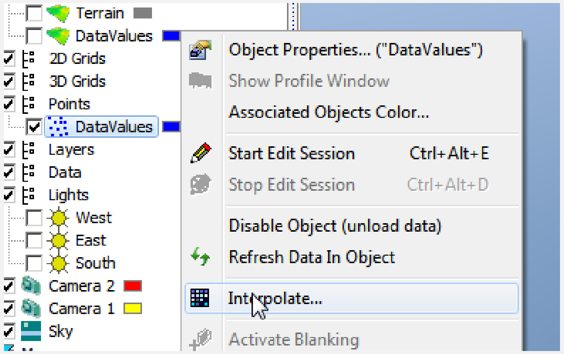
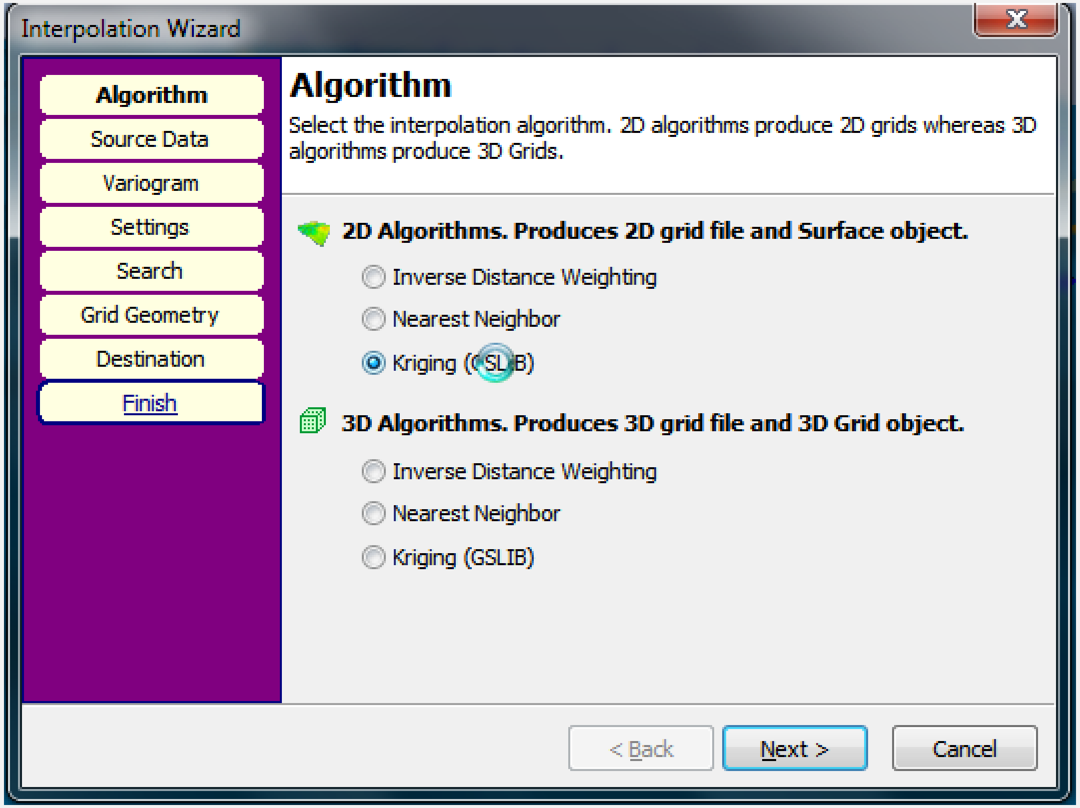
GeoScene3D - Interpolation

1. Choose you data for interpolation

2. Choose interpolation algorithm

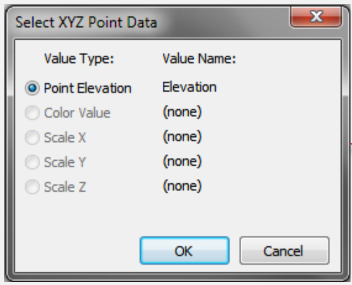


Activate



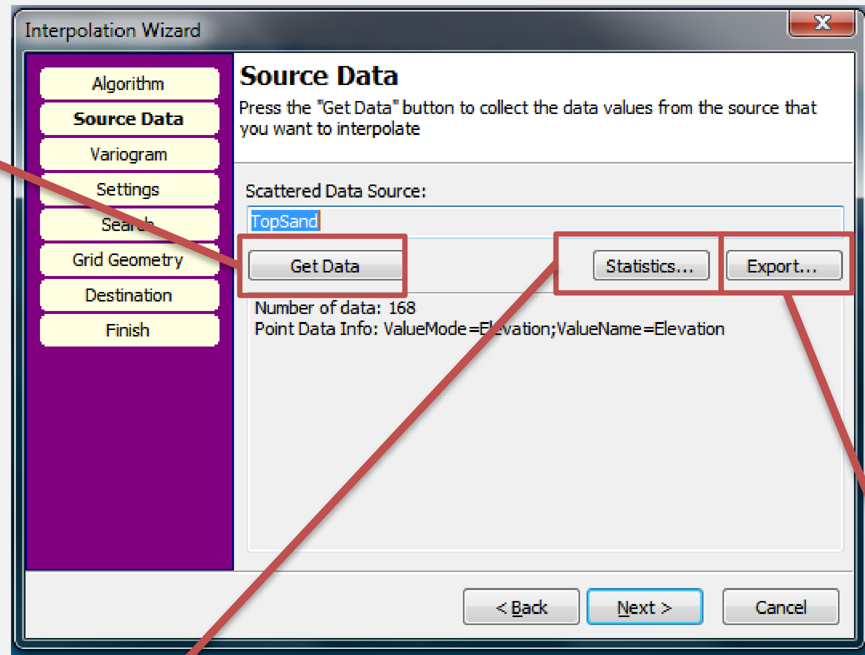
GeoScene3D - Interpolation

3: Select and inspect data…

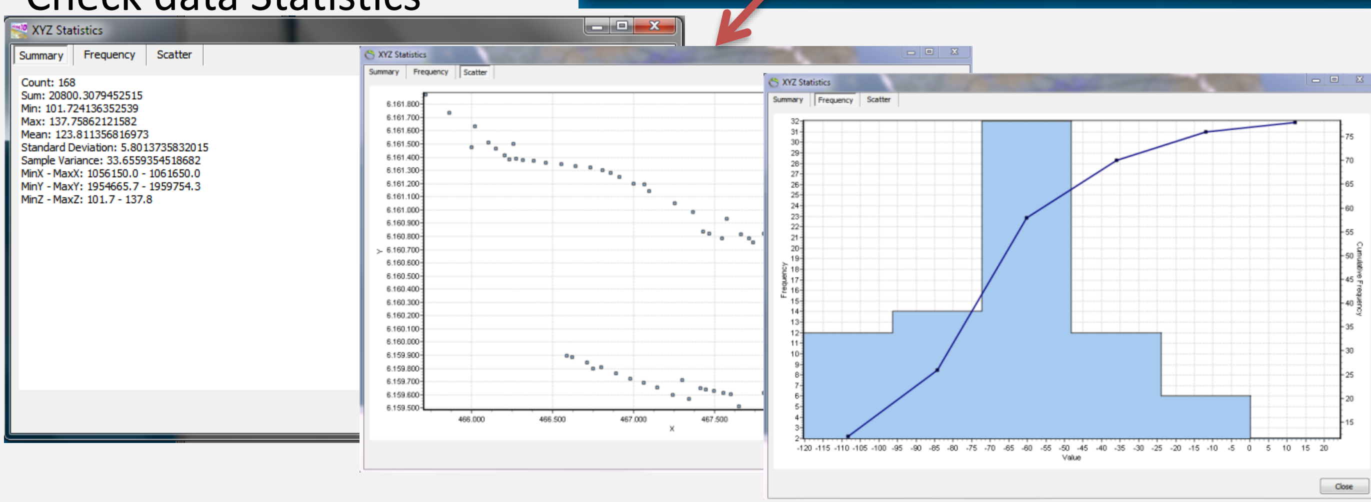


Select value to interpolate

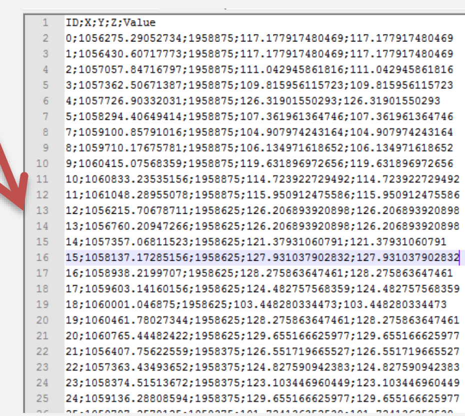
Source Data Window



Check data Statistics

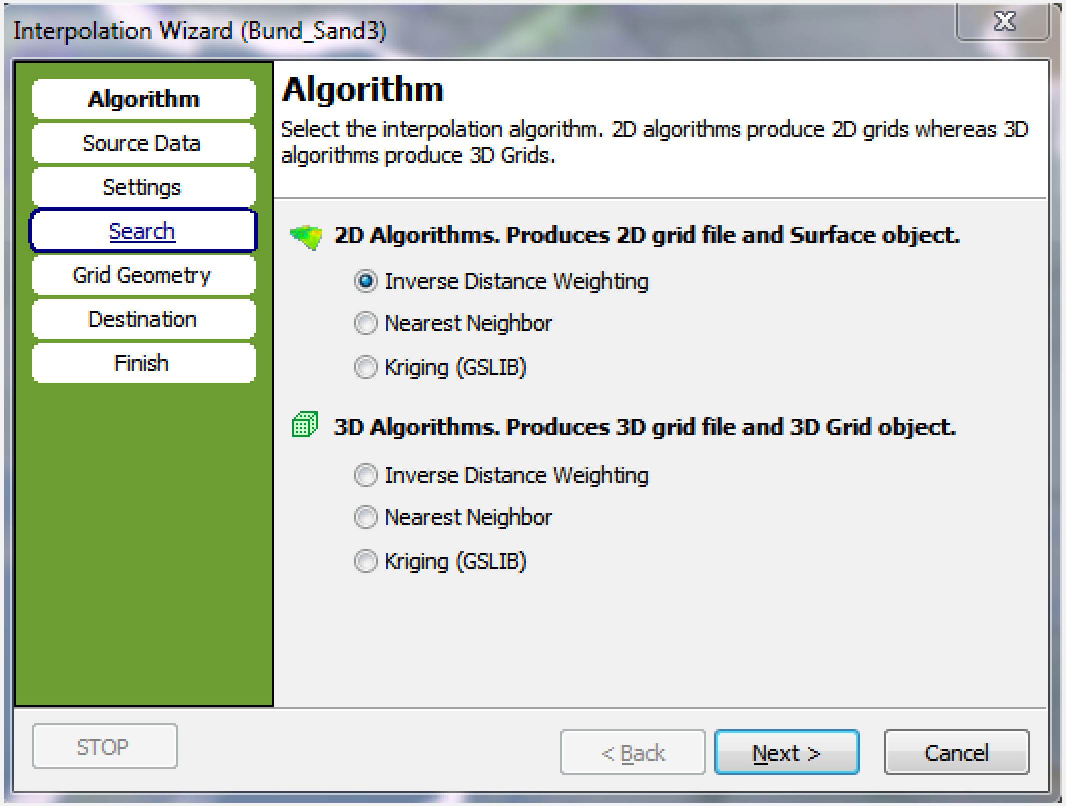


Export to CSV file



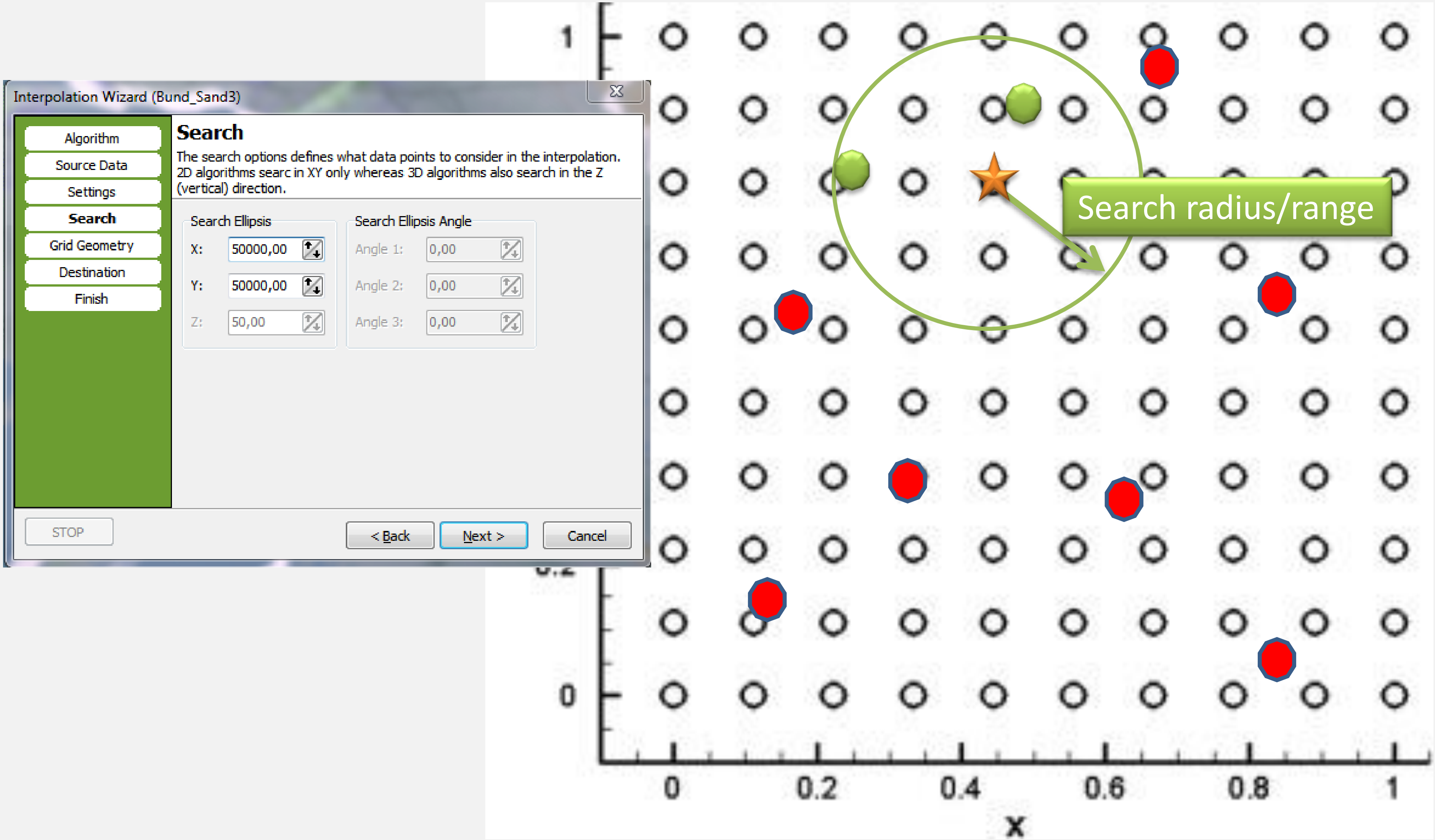
GeoScene3D - Interpolation

4: Specify settings for the algorithm selected…

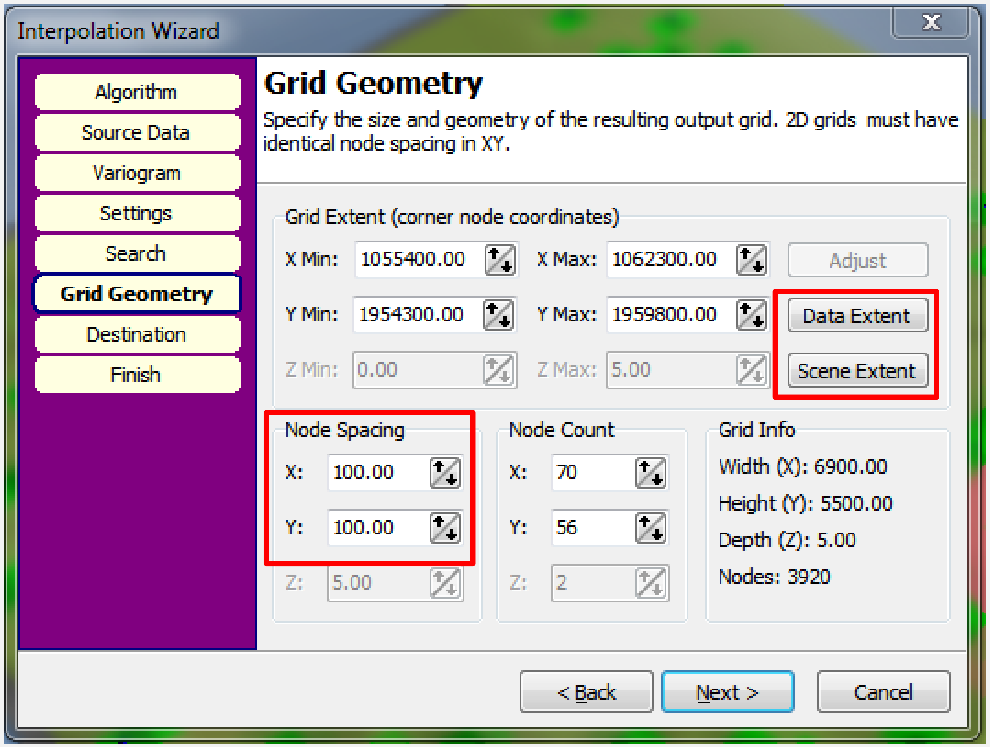


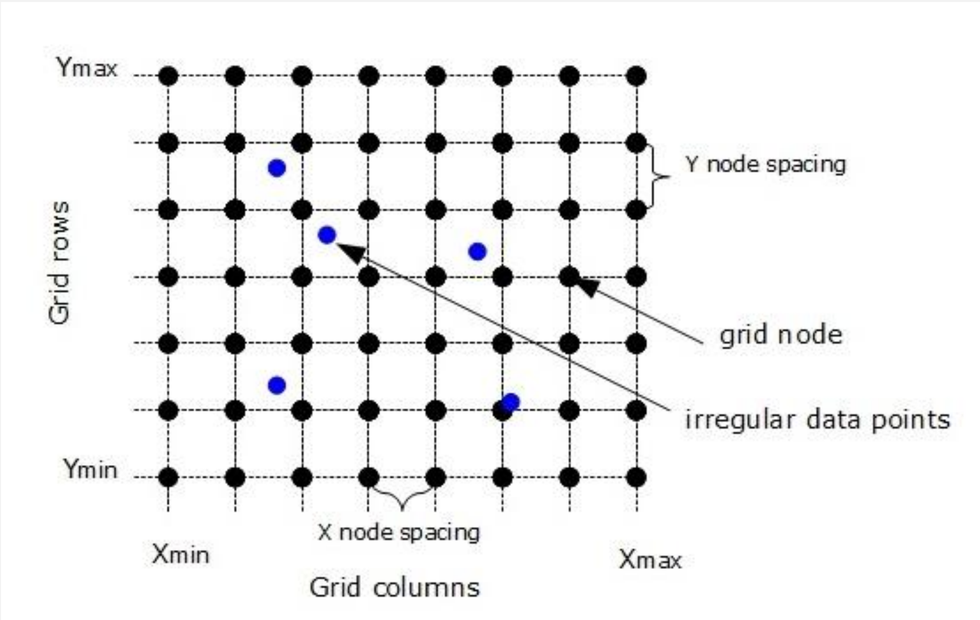
GeoScene3D - Interpolation

5: Define Search Range…



Grid Geometry and Data Extent



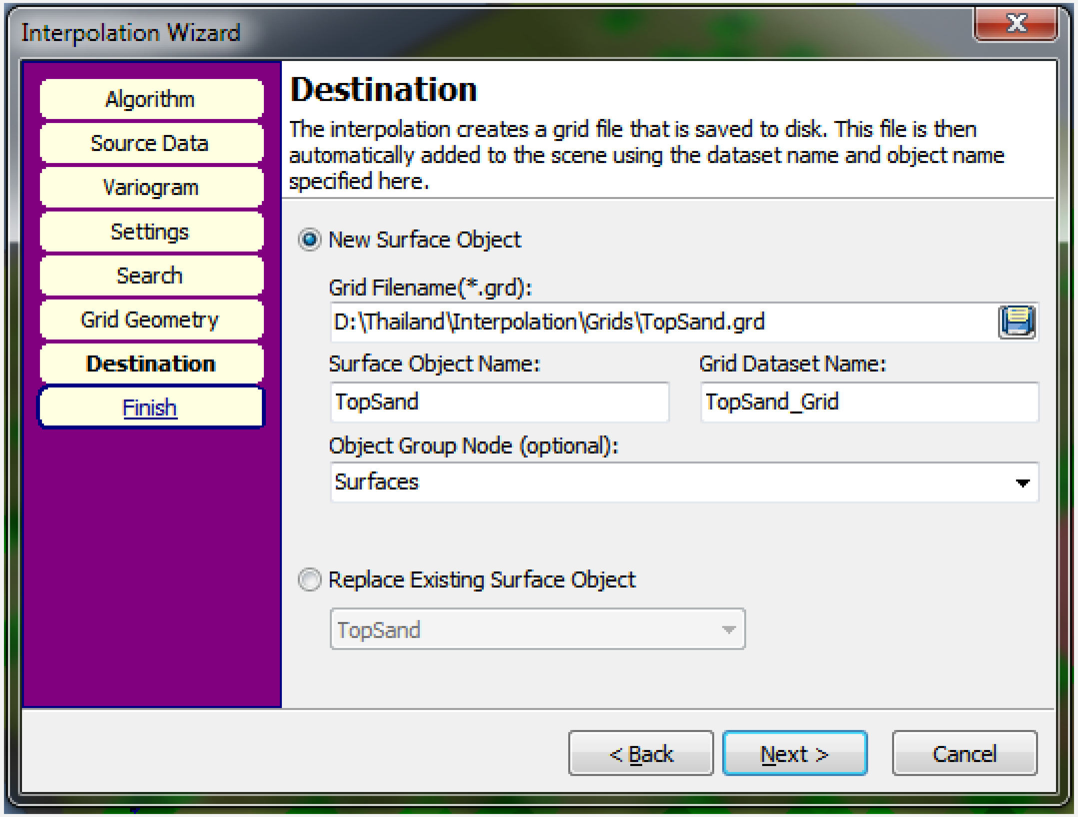


Define Resulting Grid

- How detailed features do you wish (or are able) to resolve ?
- WARNING: Small grid cells potentially leads to very large grids !
- Be conservative!

GeoScene3D - Interpolation

Destination



- Save in a project folder
- Be consistent !
- Avoid clutter in your project structure !

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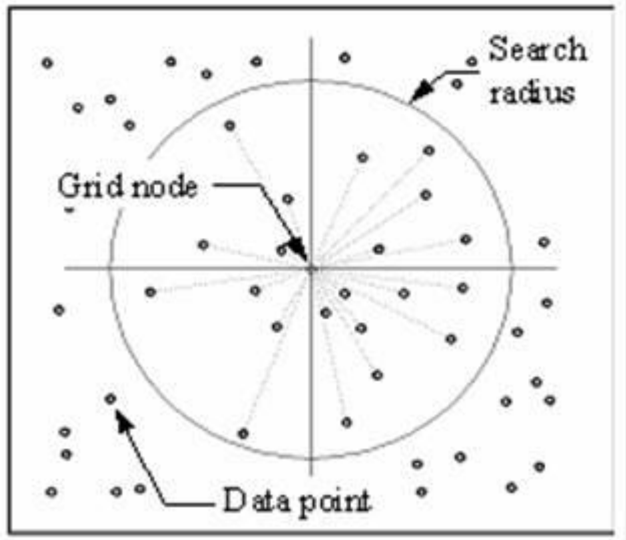
Nearest Neighbour

Description
At each grid point a value is assigned the value of the nearest data point within the search radius.

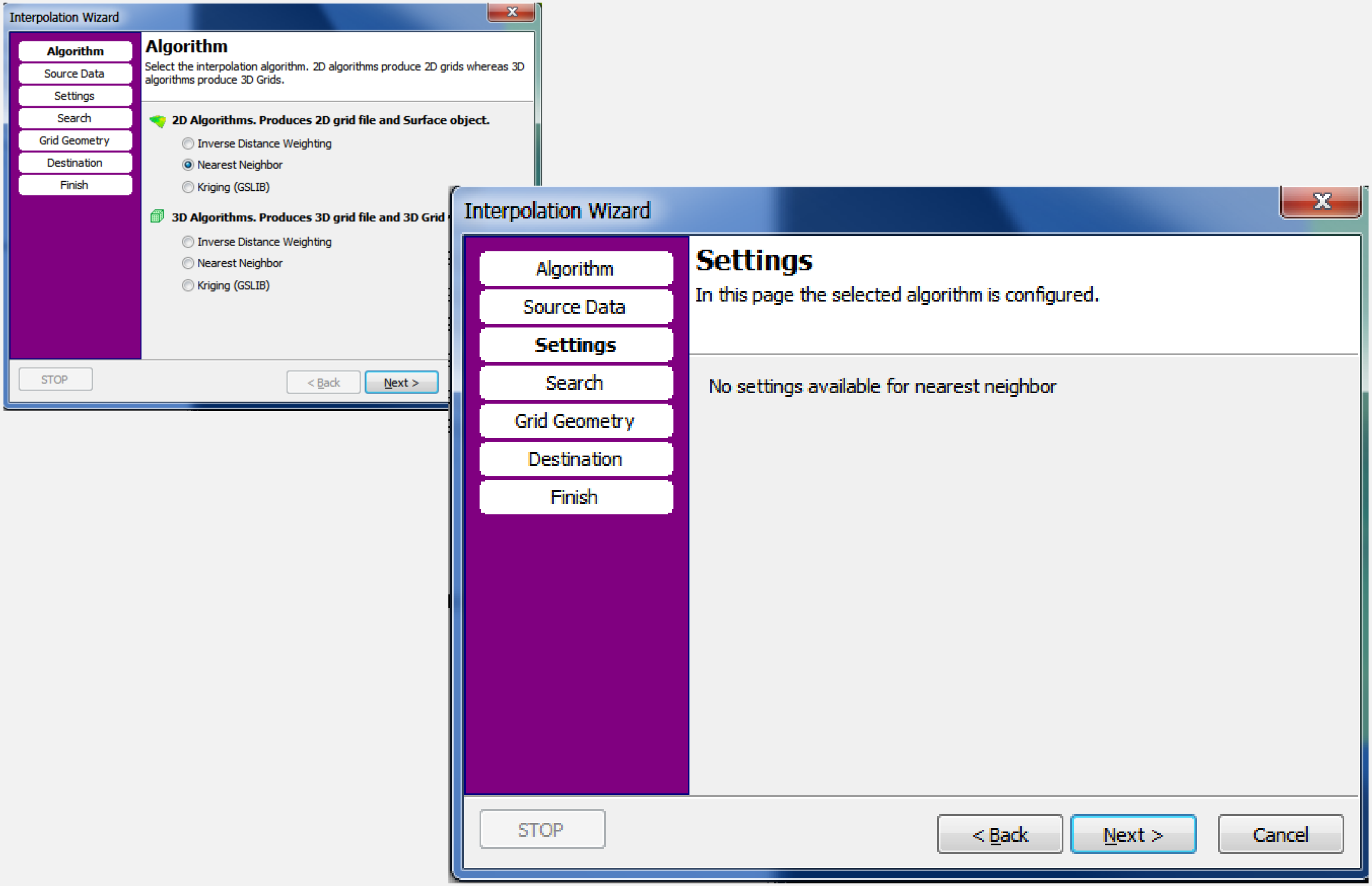


Parameters
- None

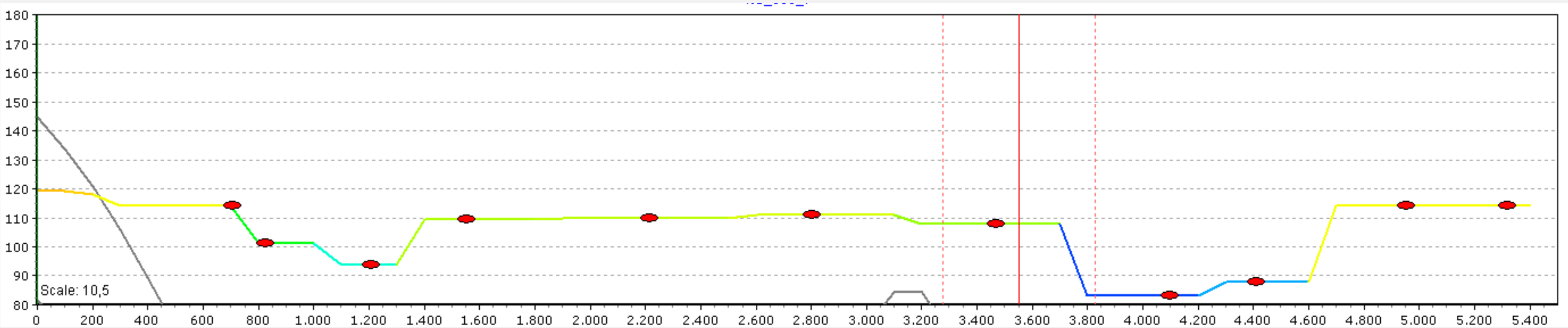
Advantages
- Simple and fast
- Keeps high and low extreme values in the interpolated surfaces.

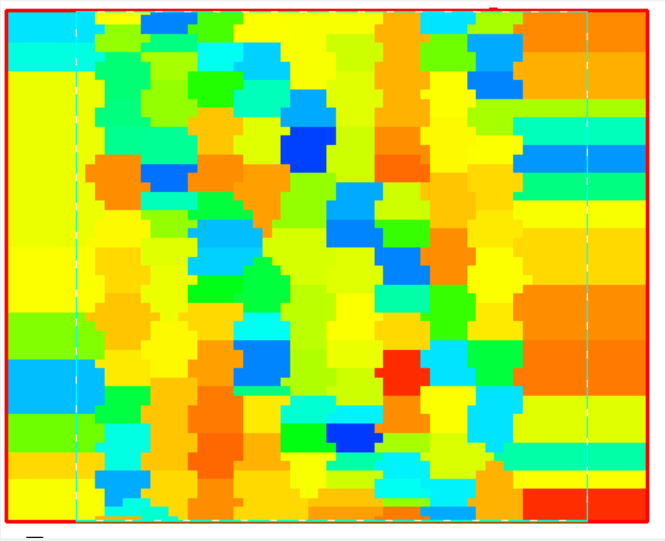
Disadvantages
- Maybe to simple ☺
- Results in high frequency “noisy” surface

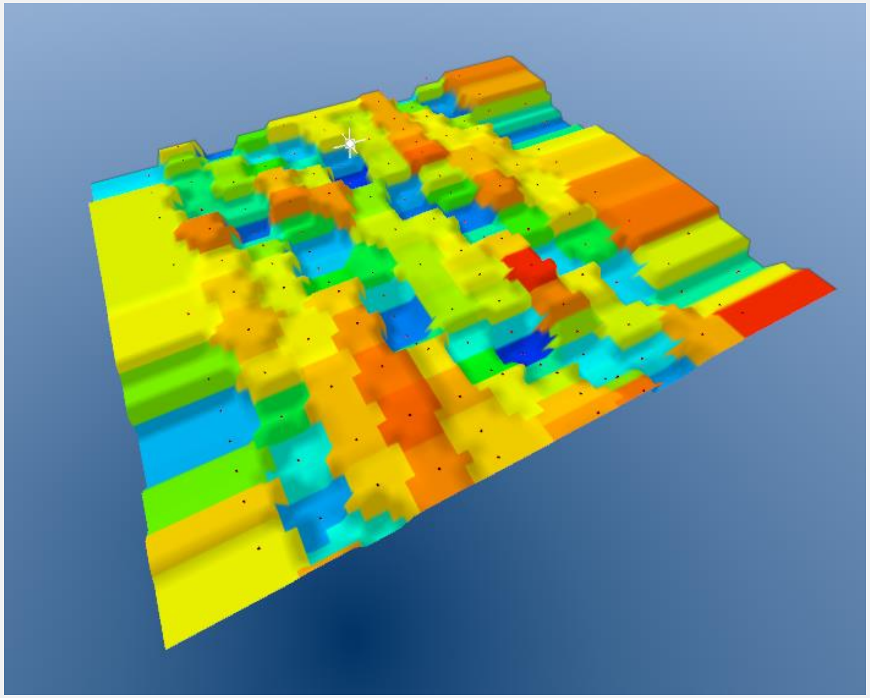
Nearest Neighbour in GeoScene3D



GeoScene3D - Interpolation







When would this be an appropriate way to interpolate data?

Agenda

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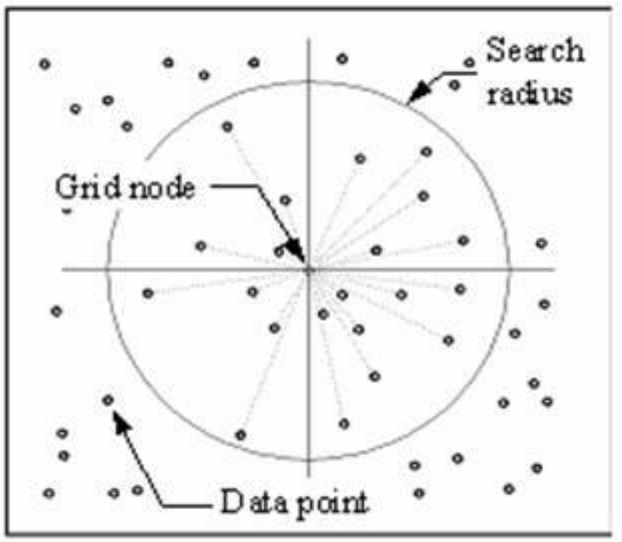
Inverse Distance Weighted (IDW)

Description

At each grid point a value is assigned based on a *weighted mean* of the values within the search radius. The weighting is done by distance lifted to a Power.



Parameters

- Search Radius
- Exponent (Power)
- Smooth factor
- Search direction
- Point Limits / Quadrant Search

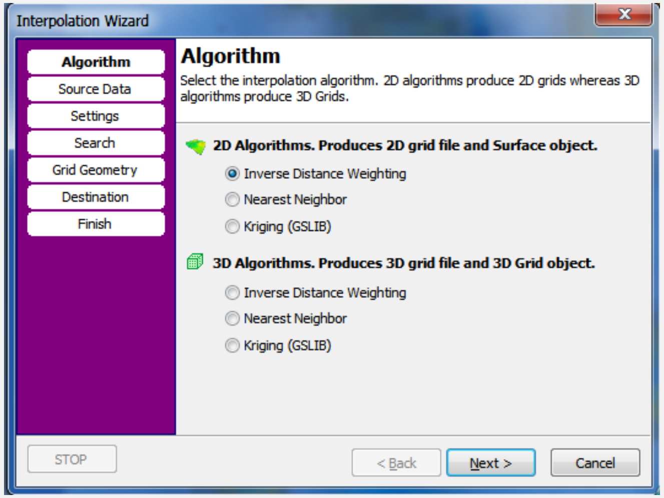
Advantages

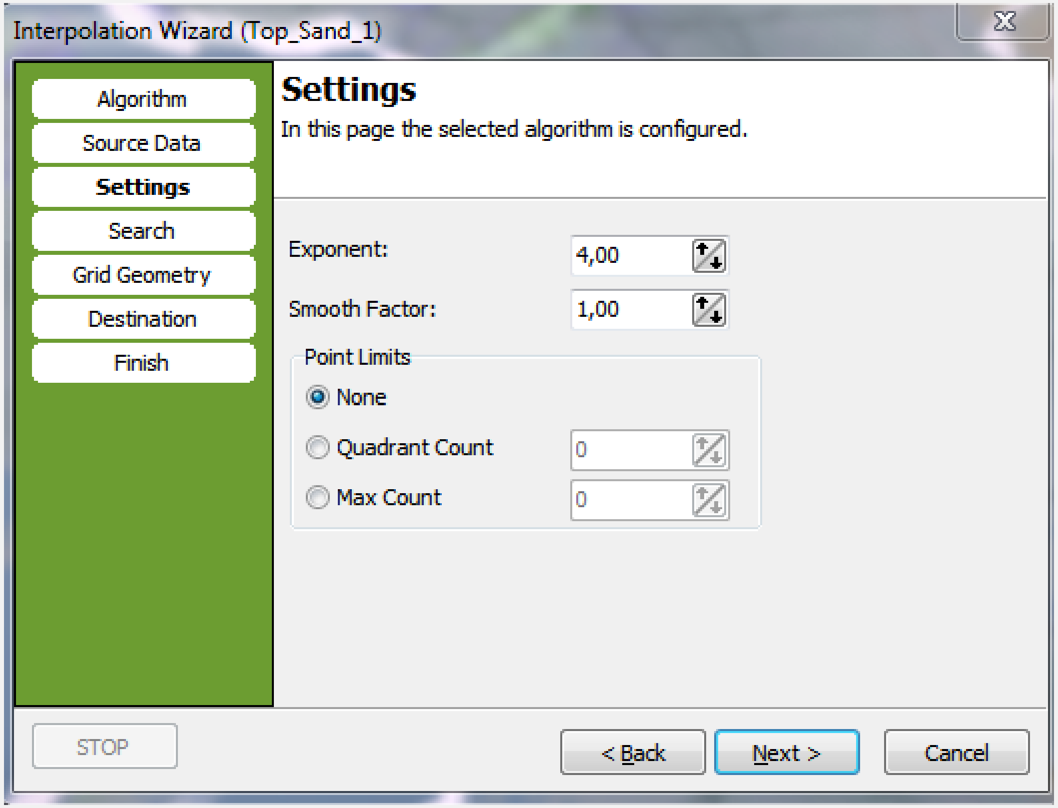
- Simple and relatively fast
- Gives nice smooth picture

Disadvantages

- Smoothing removes extremes from data

Inverse Distance Weighted in GeoScene3D





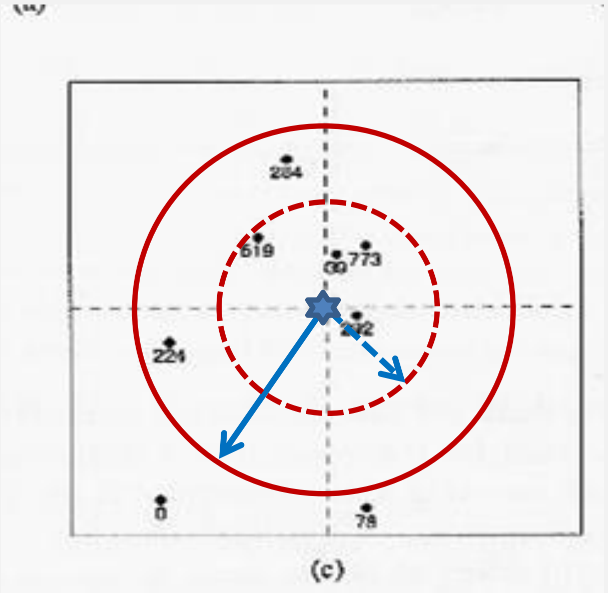
Inverse Distance Weighted in GeoScene3D

Max Count

- Defines a limit of max number of data points within a defined search radius. A kind of a “dynamic search radius”

Example:

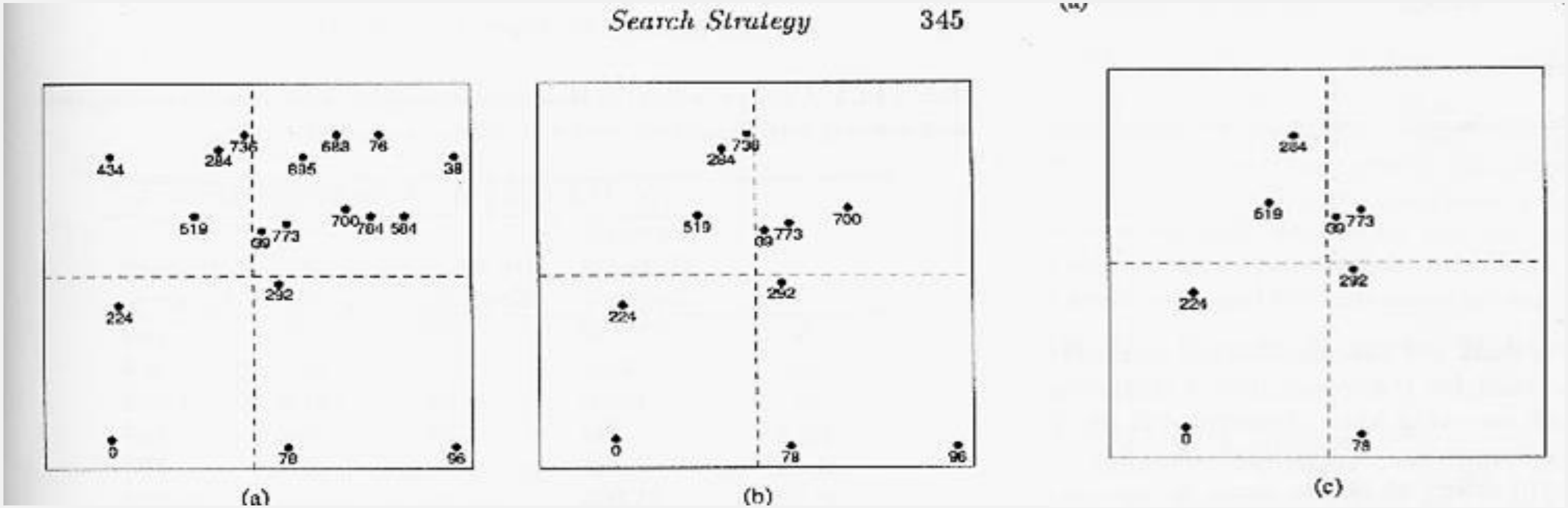
- Search Radius = 250 m
- 6 points within Search Radius
- Max Count = 4
- ”Real” Search Radius will be smaller than 250 m (approx. 180 m)



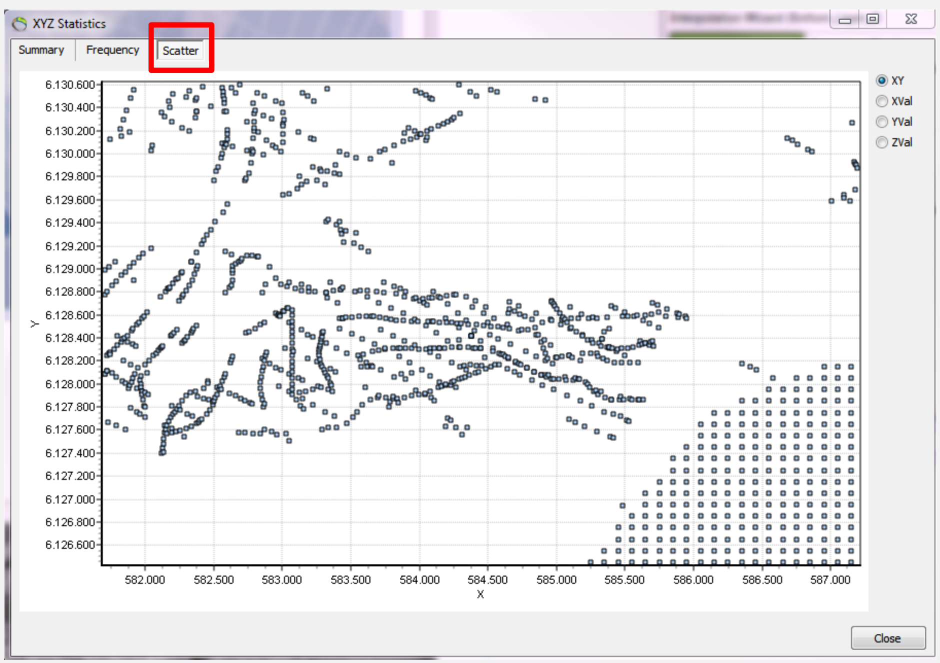
Inverse Distance Weighted in GeoScene3D

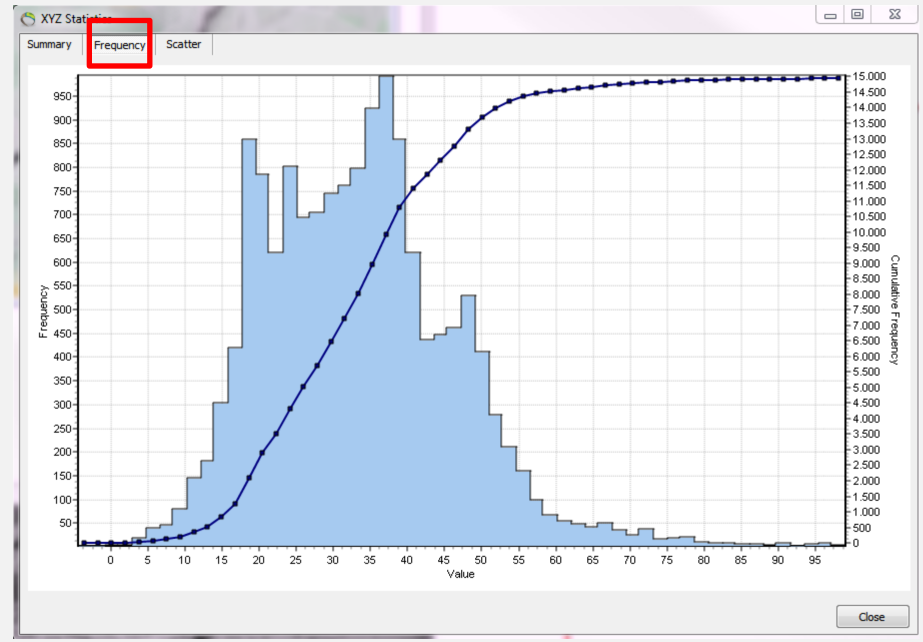
Quadrant Search

- Divide the search neighborhood into 4 quadrants and specify the maximum number of samples per quadrant
- If a quadrant has samples less than the maximum, it keeps all samples. Otherwise keeps the closest ones
- Quadrant search accomplishes some declustering



Interpolation – distribution of data points



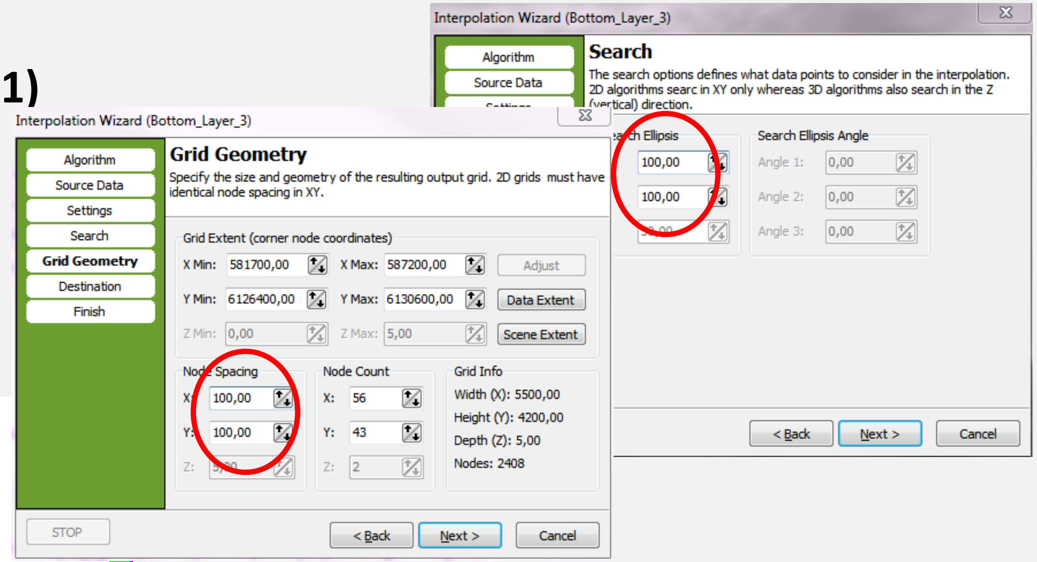


GeoScene3D - Interpolation

Interpolation – Inverse Distance (example 1)

SR = 100

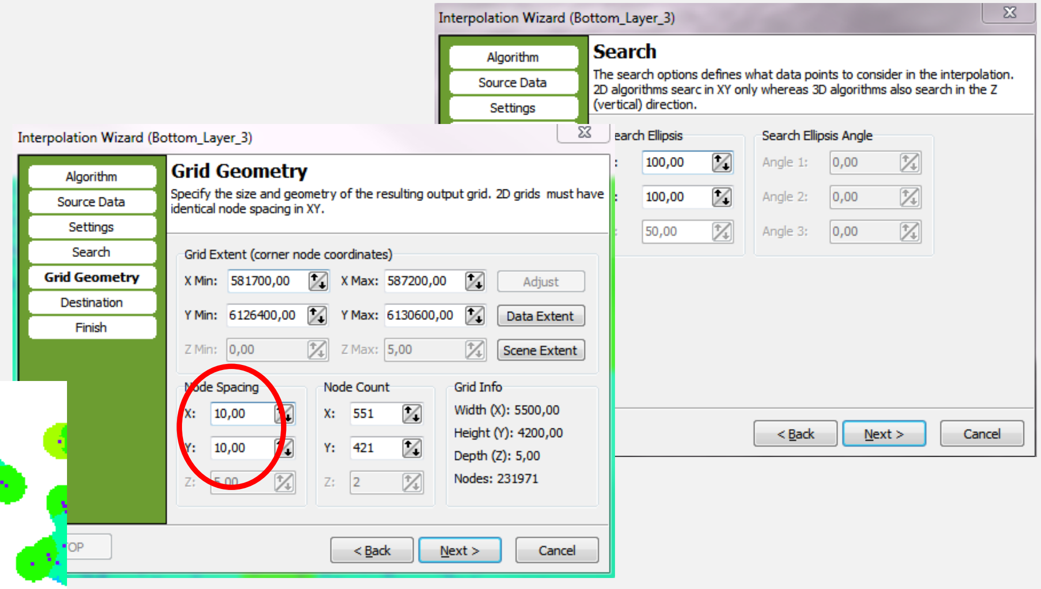
Cell Size = 100

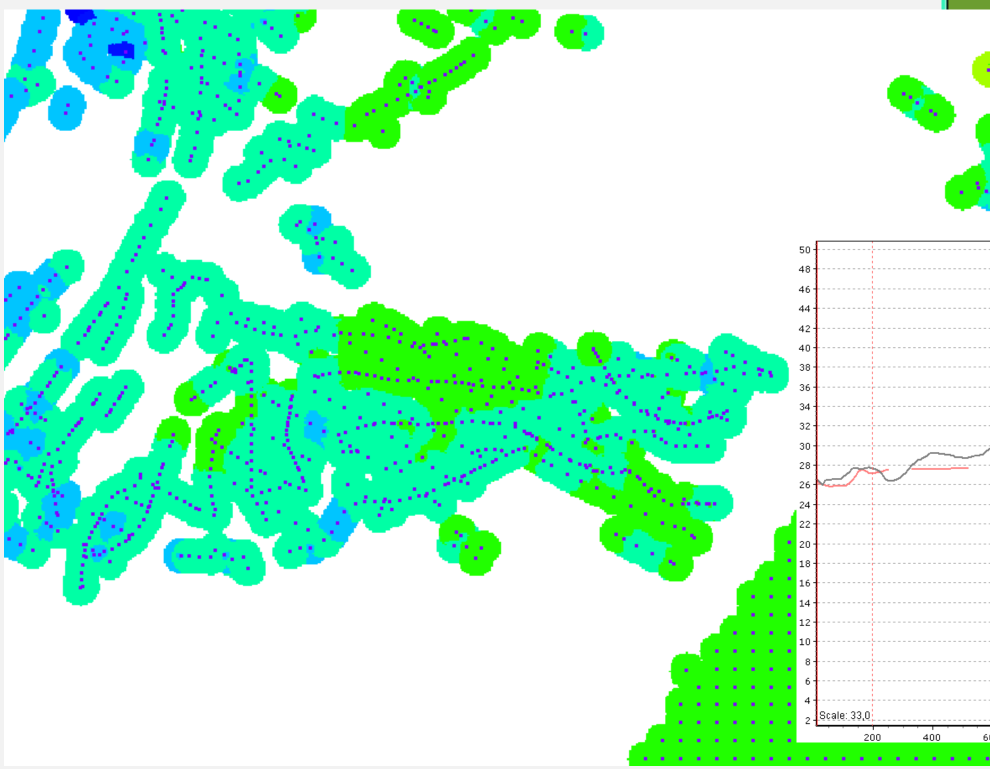


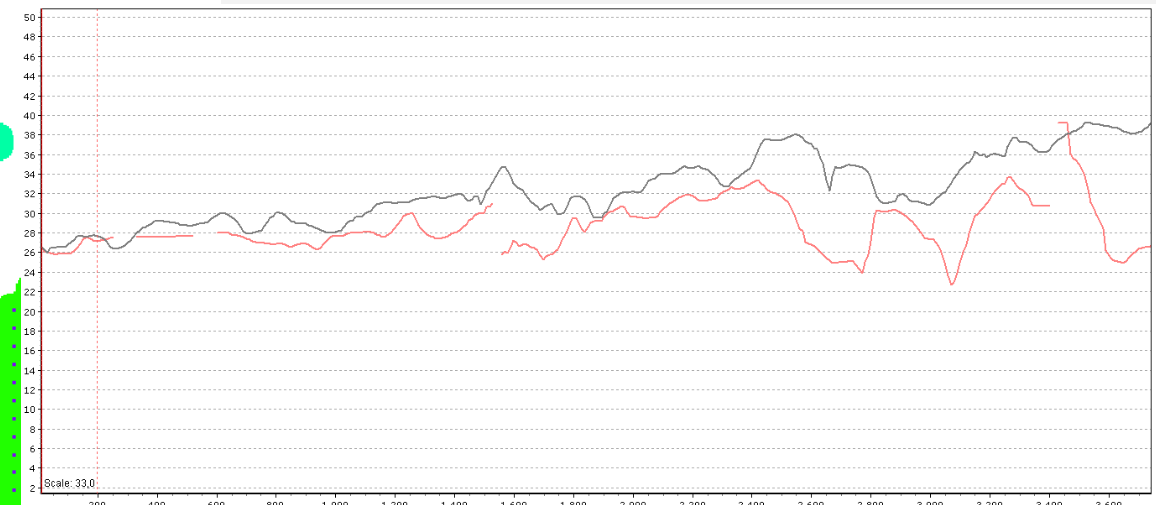
GeoScene3D - Interpolation

GEOScene3D ...by I•GIS

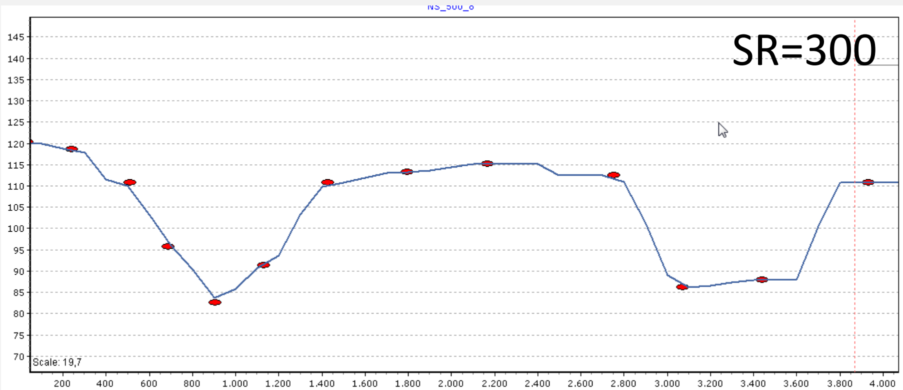
Interpolation – Inverse Distance (example 2)
SR = 100
Cell Size = 10

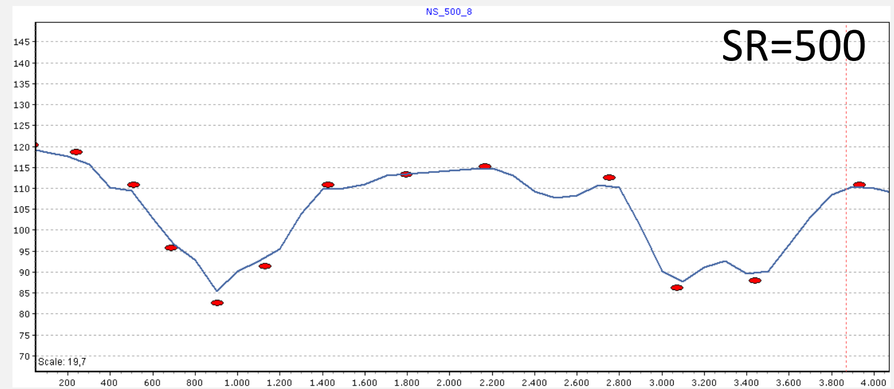


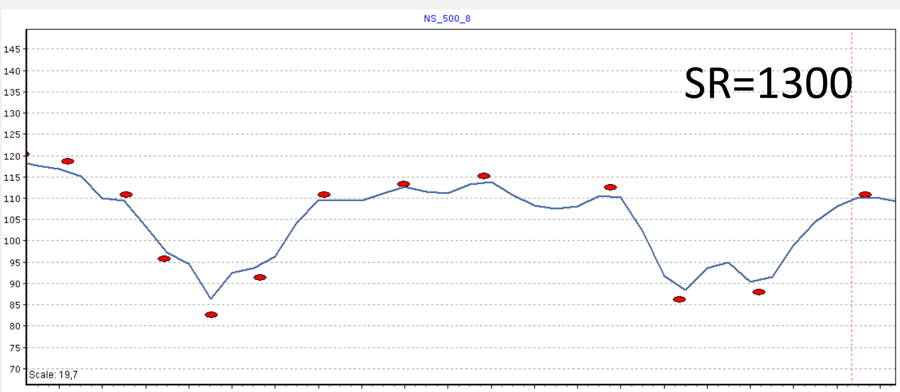




Interpolation – Inverse Distance: *Search Radius*







$$w_i(x) = \frac{1}{\left(\sqrt{(x-x_i)^2+(y-y_i)^2+s^2}\right)^p}$$

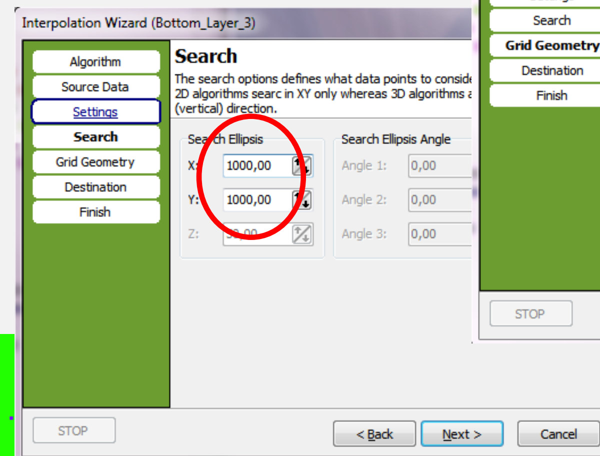
- As search radius increases more data are included in the interpolation
- Weighting has an exponential growth as we approach the location of data points
- Smoothing counteracts this…

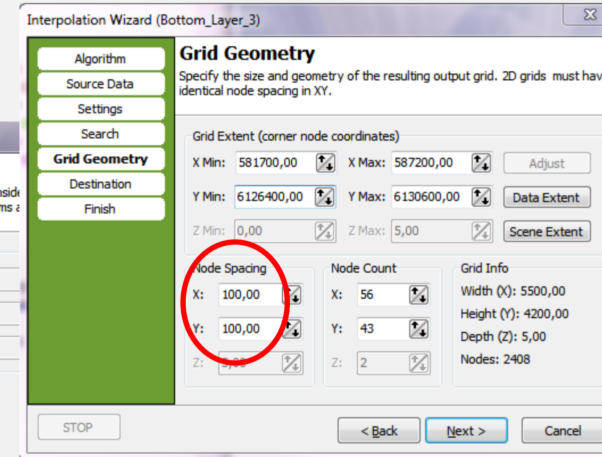
Interpolation – Inverse Distance (example 3)

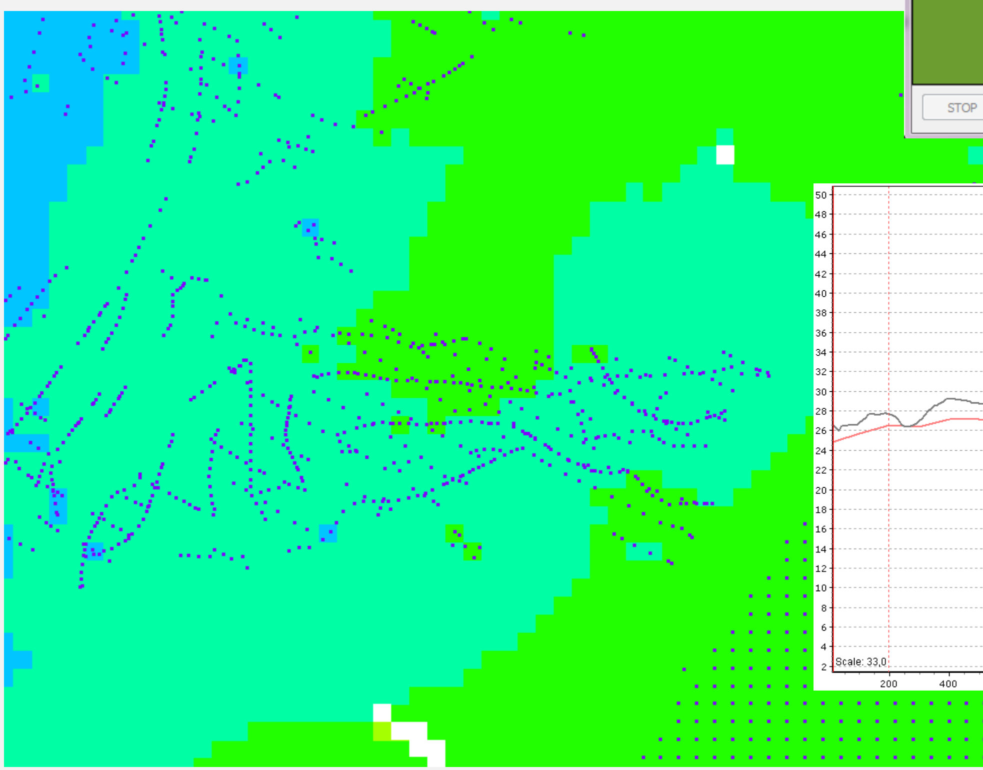
SR = 1000

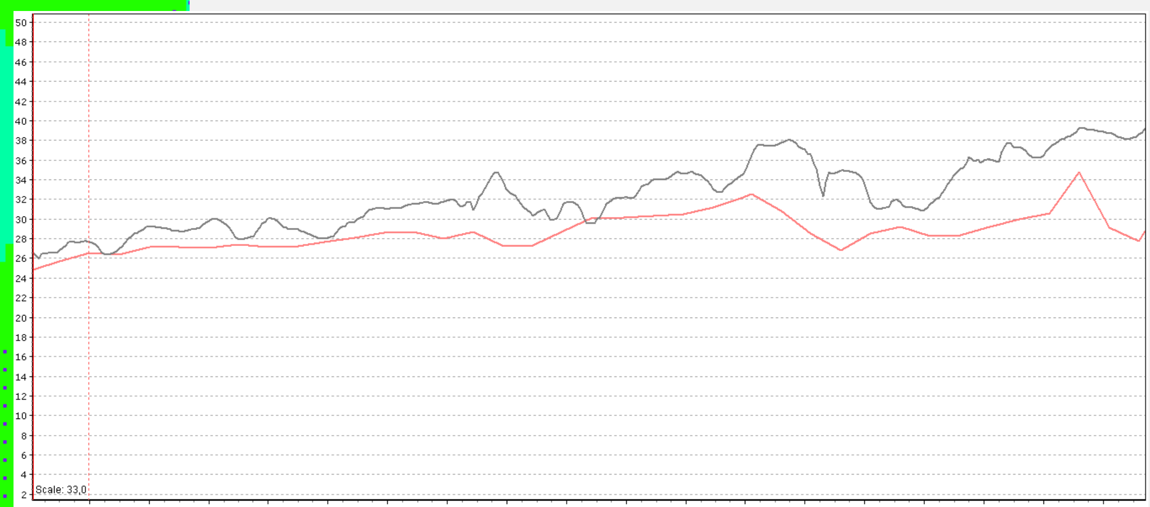
Cell Size = 100

Point Limit = None



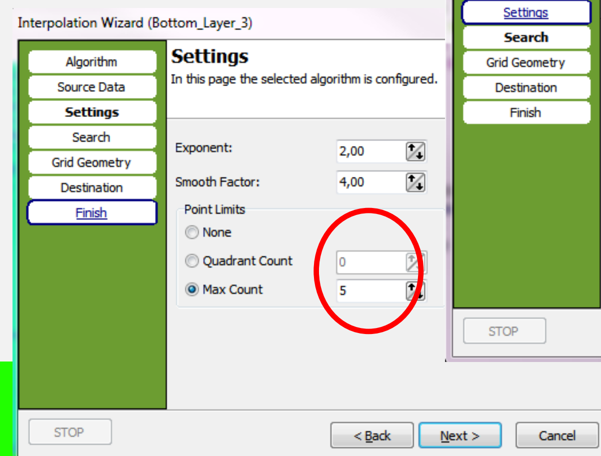


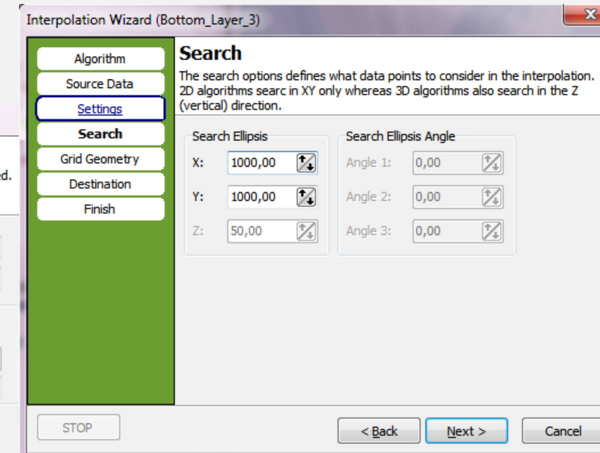


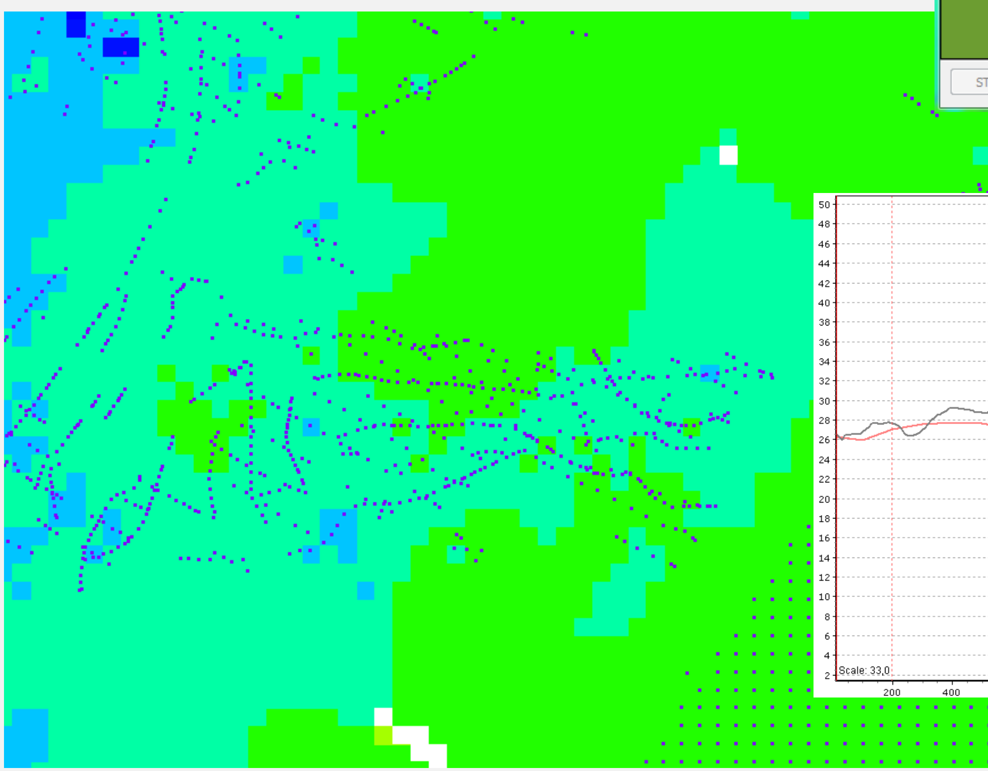


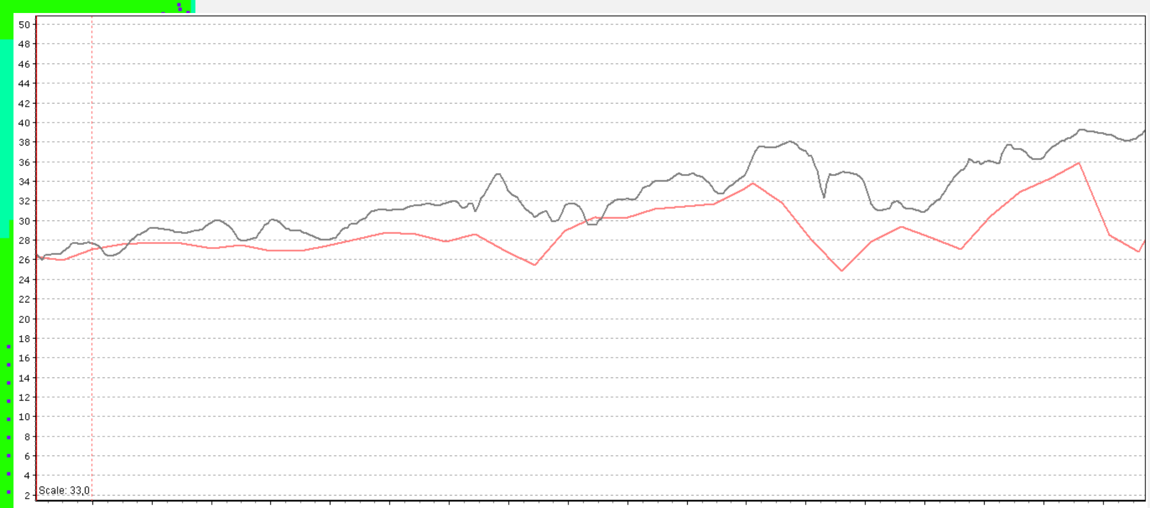
GeoScene3D - Interpolation

Interpolation – Inverse Distance (example 4)
SR = 1000
Cell Size = 100
Point Limit = 5









GeoScene3D - Interpolation

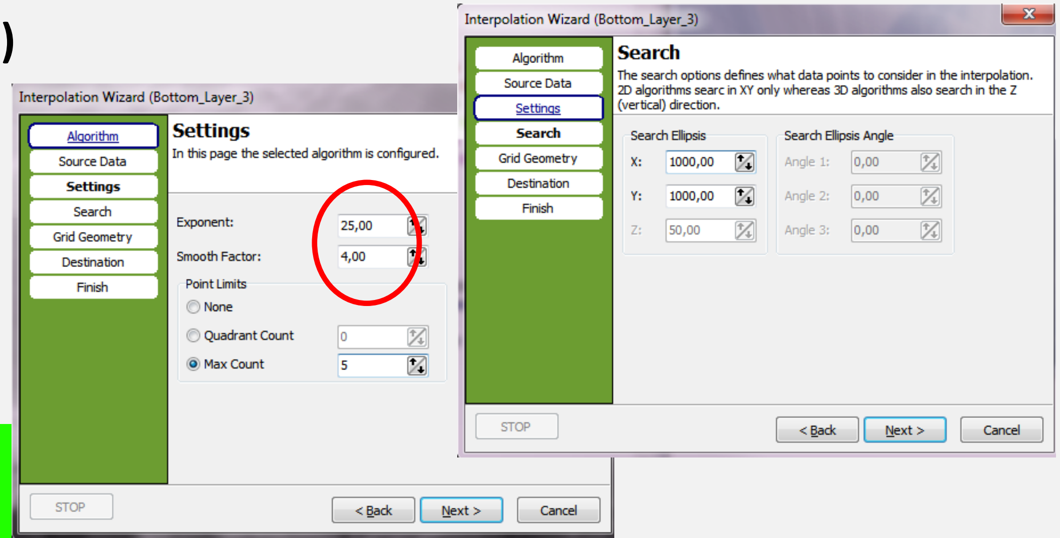
Interpolation – Inverse Distance (example 5)

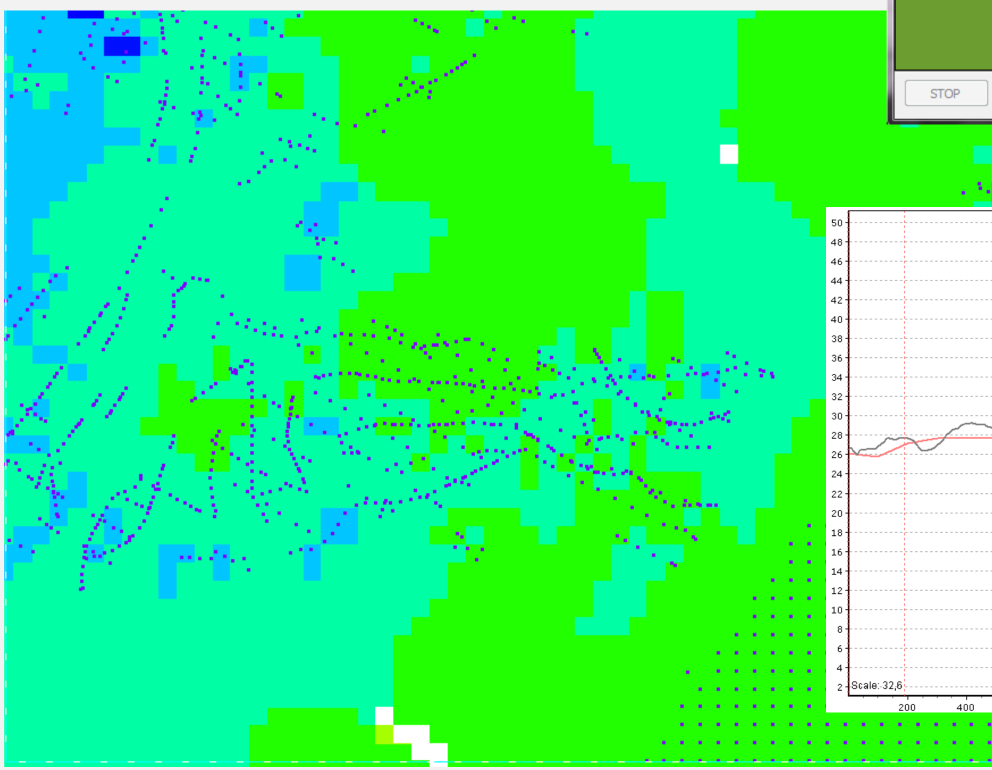
SR = 1000

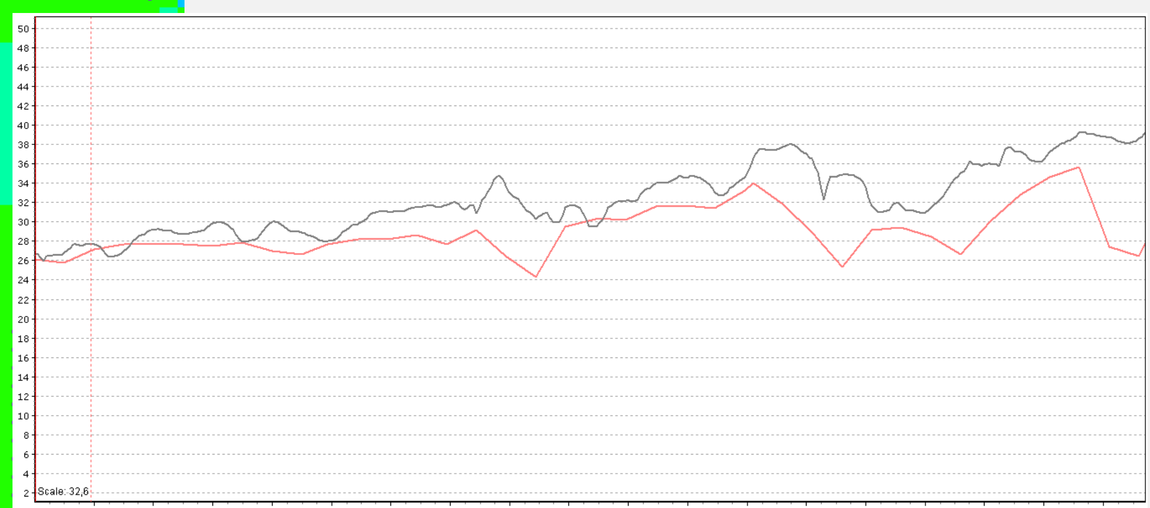
Cell Size = 100

Point Limit = 5

Exponent = 25







Interpolation – Inverse Distance: *Smooth Factor*

IDW General definition:

Interpolate a value, u, at a given point **x** based on samples for $u_i = u(x_i)$ for $i = 0, 1, ..., N$

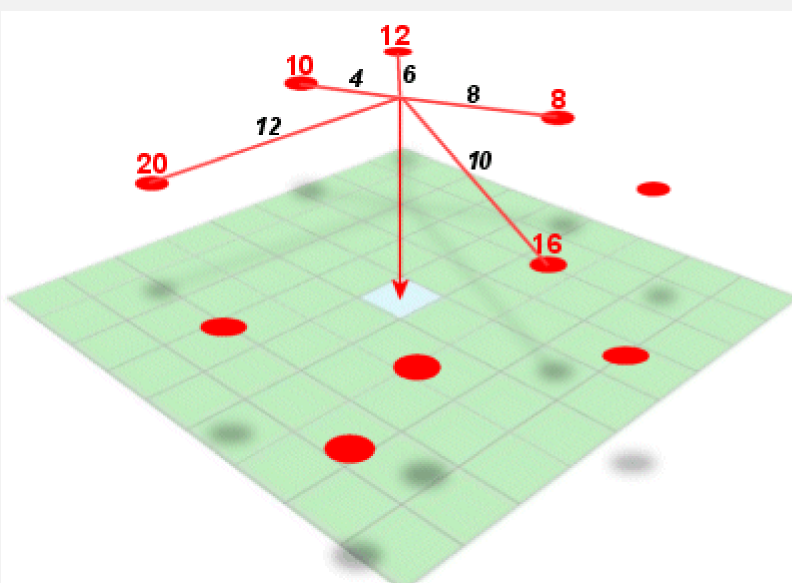
$$u(\mathbf{x}) = \sum_{i=0}^{N} \frac{w_i(\mathbf{x}) u_i}{\sum_{j=0}^{N} w_j(\mathbf{x})},$$

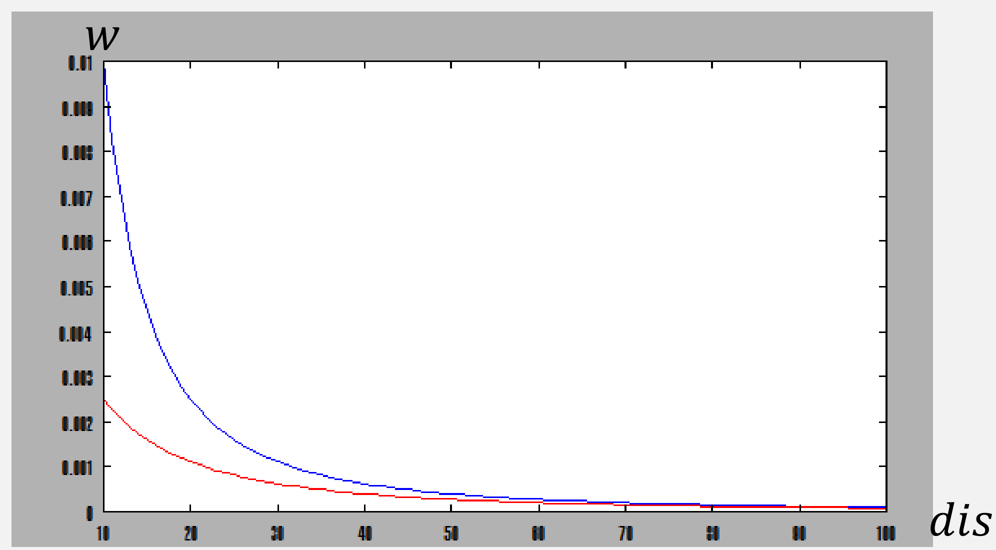
where weighting function is: $w_i(\mathbf{x}) = \frac{1}{d(\mathbf{x}, \mathbf{x}_i)^p}$

In GeoScene3D, a smoothing factor, s, is introduced in the weighting function as:

$$w_i(x) = \frac{1}{(\sqrt{(x - x_i)^2 + (y - y_i)^2 + s^2})^p}$$

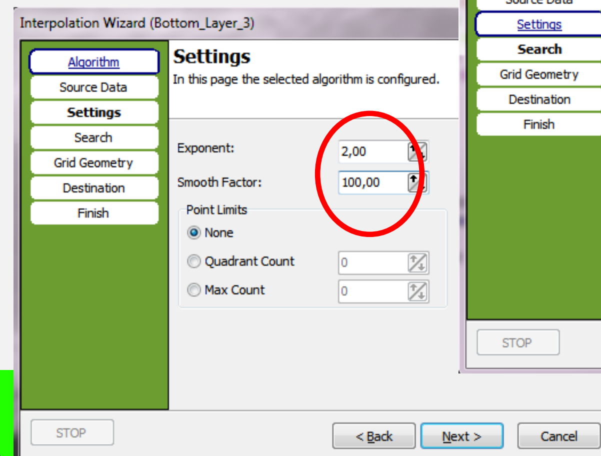
S: Makes weight smaller close to x.

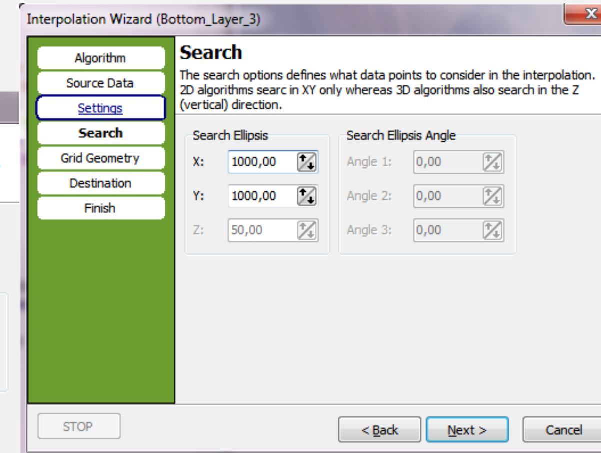


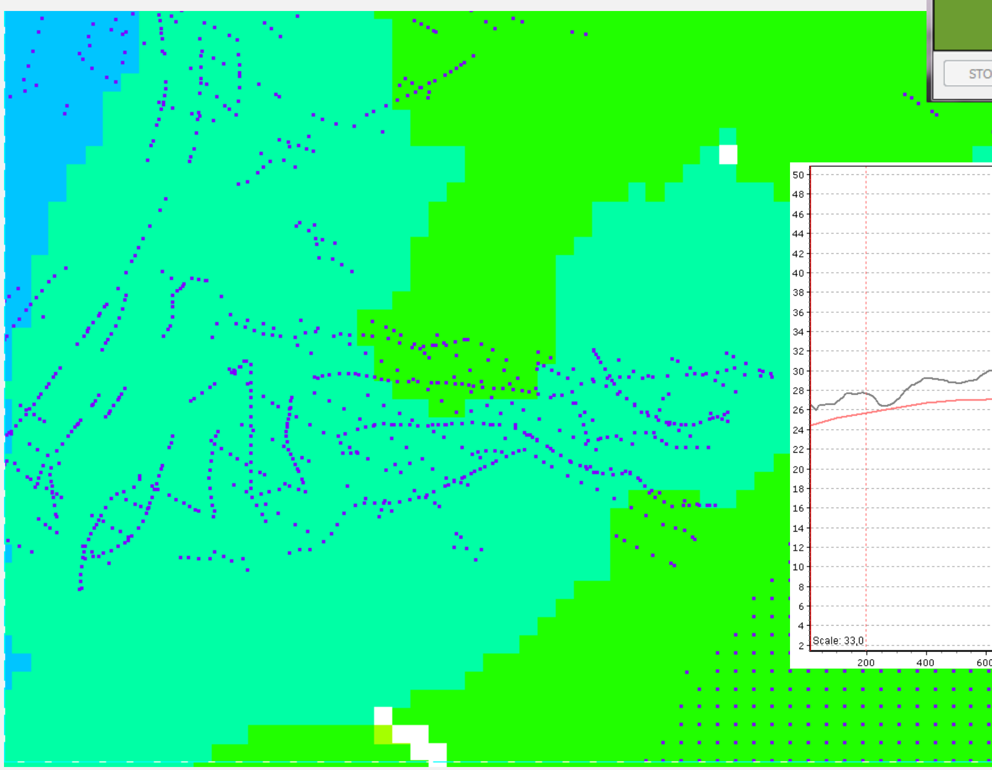


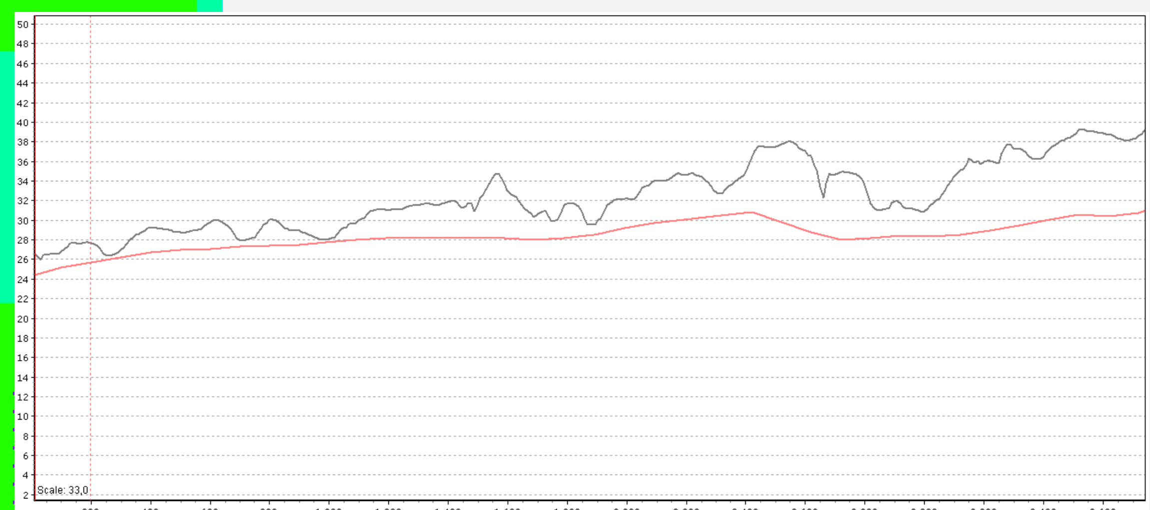
GeoScene3D - Interpolation

Interpolation – Inverse Distance (example 6)
SR = 1000
Cell Size = 100
Point Limit = None
Smooth Factor = 100

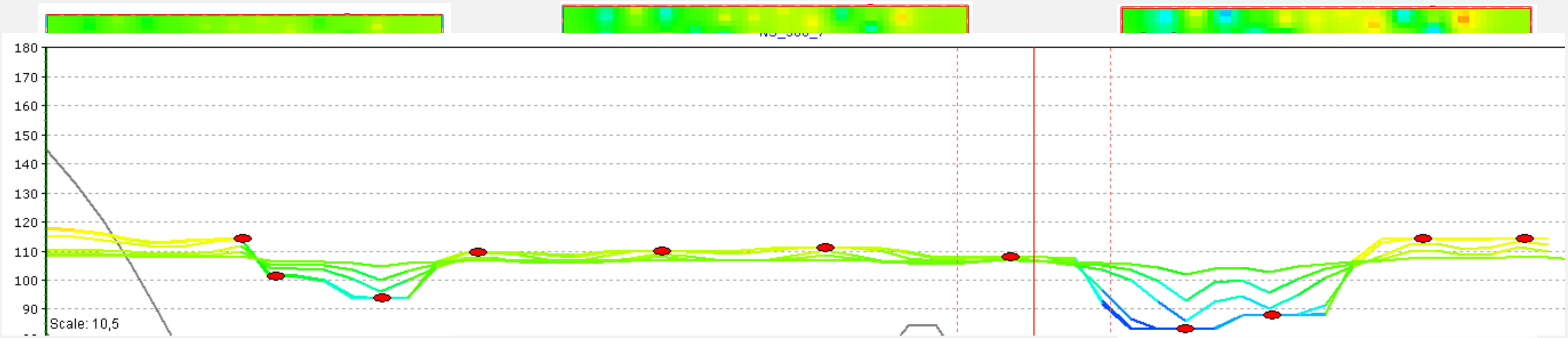


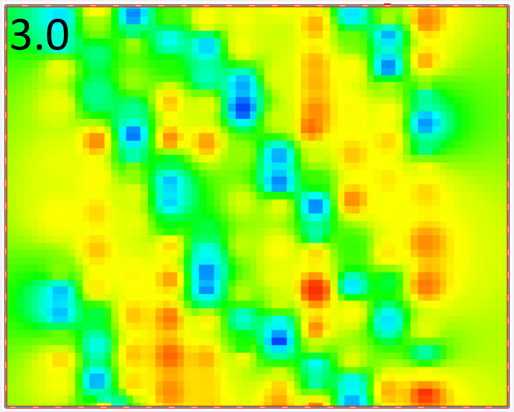


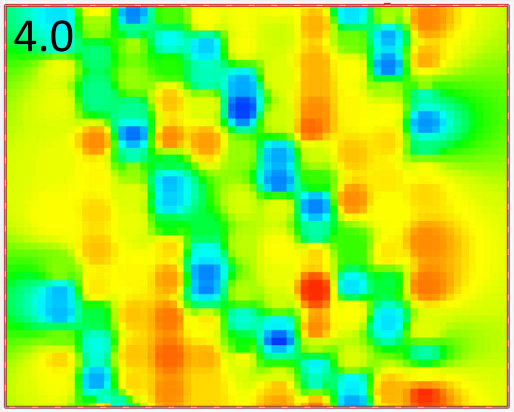


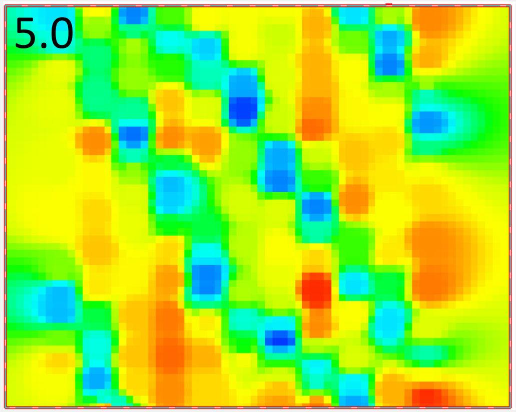


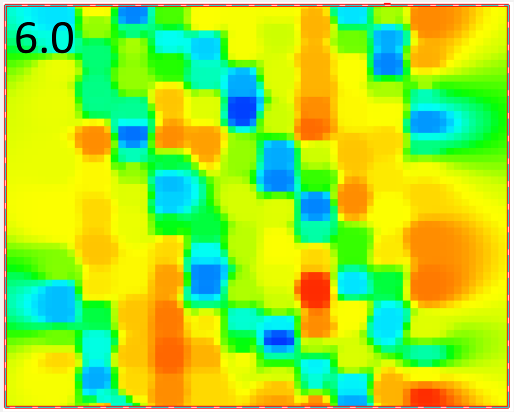
GeoScene3D - Interpolation

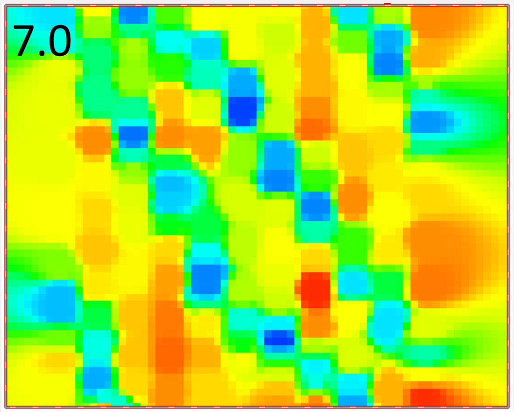






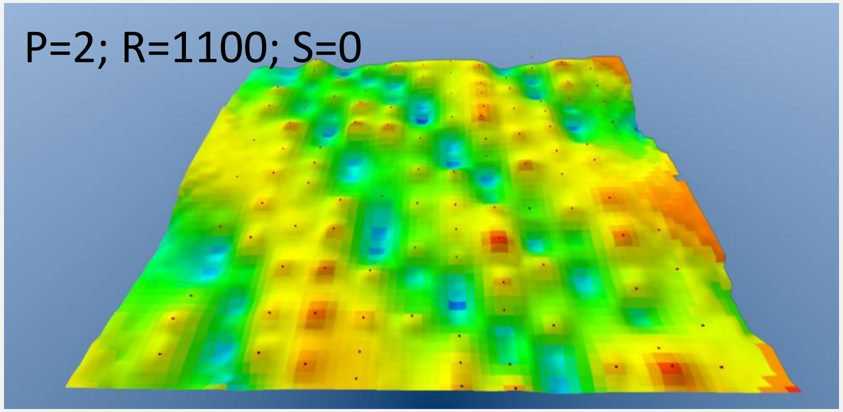


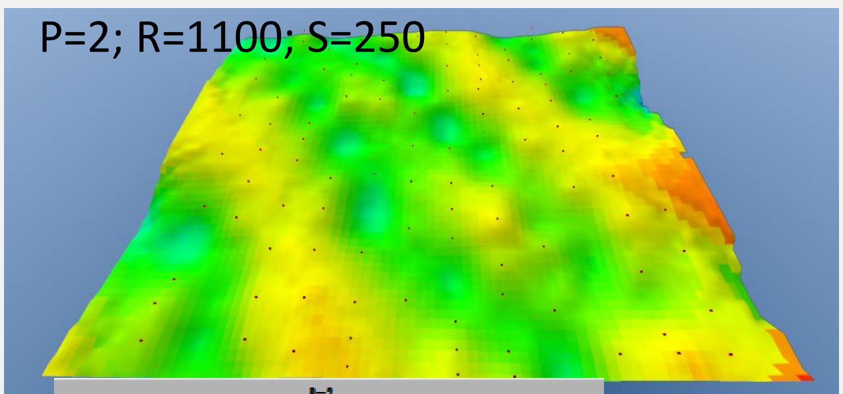


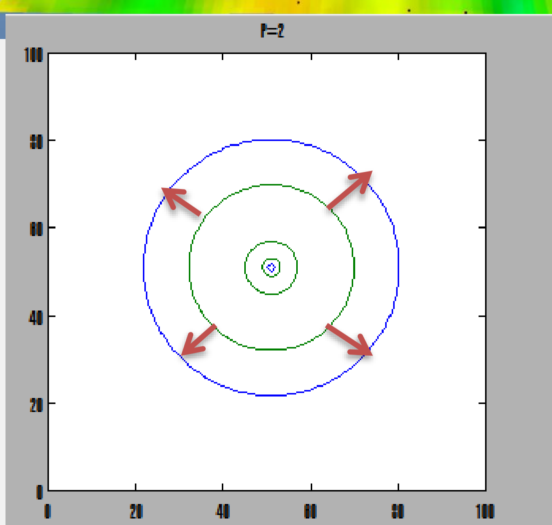


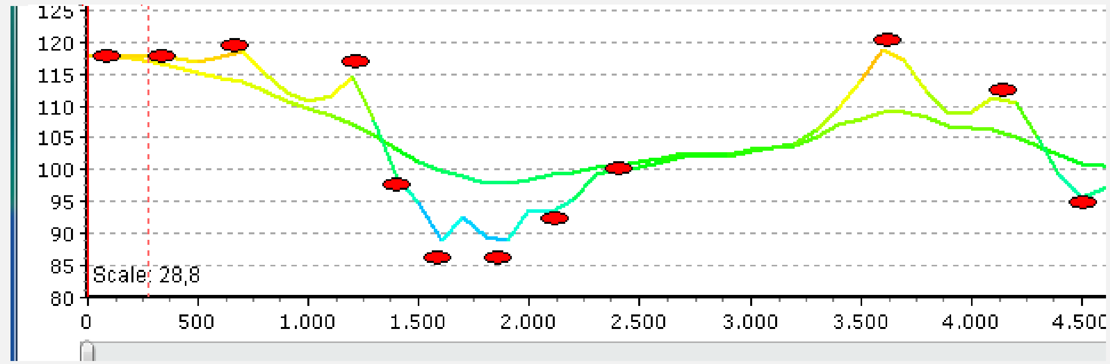
Different Exponents

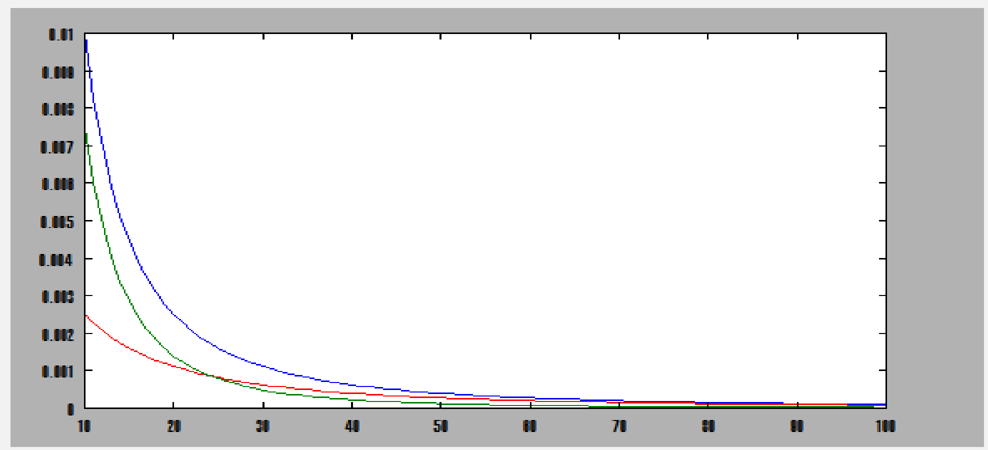
GeoScene3D - Interpolation











- Smoothing factor counteracts this – see formula:

$$w_i(x) = \frac{1}{\left(\sqrt{(x - x_i)^2 + (y - y_i)^2 + s^2}\right)^p}$$

GeoScene3D - Interpolation

Inverse Distance Weighting

- Works best for dense, evenly-spaced sample point sets.
- Does not consider any trends in the data
  - *if actual surface values change more in the north-south direction than they do in the east-west direction the interpolated surface will average out this bias rather than preserve it (Quadrant Count can to some extent compensate for this).*
- Cannot make estimates above the maximum or below the minimum sample values.
  For an elevation surface, this has the effect of flattening peaks and valleys (unless their high and low points are part of the sample).
- Interpolated value will correspond to data value in data point – if grid and point are overlapping.
- Smooth Factor counteracts spiking…
- Check your search radius if experiencing interpolation artifacts…
- Check your data distribution in the statistics – high differences might call for a higher Exponent…

Agenda

- Interpolation defined
- General workflow in GeoScene3D
- Interpolation methods: Nearest-Neighbour (NN)
- Interpolation methods: Inverse Distance Weightning
- Interpolation methods: Kriging
- Summing up

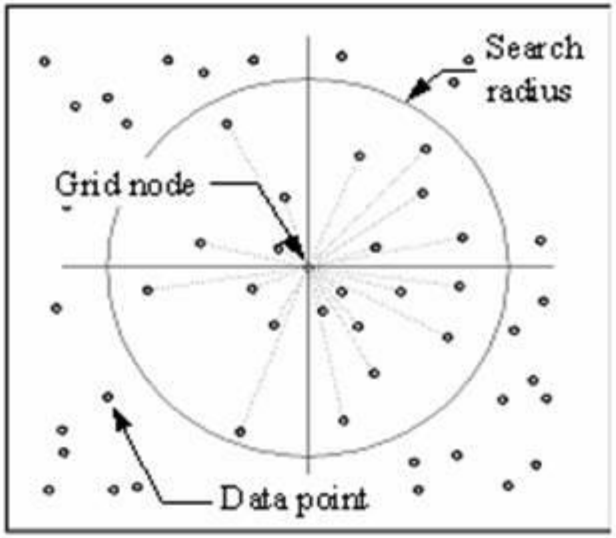
Kriging

Description

At each grid point a value is assigned based on a weighted using a statistical approach. Uses GSLIB.

Parameters

- MANY !! ☺



Advantages

- Based on statistics
- Supplies a measure of uncertainty – variance grid.
- A statistical approach

Disadvantages

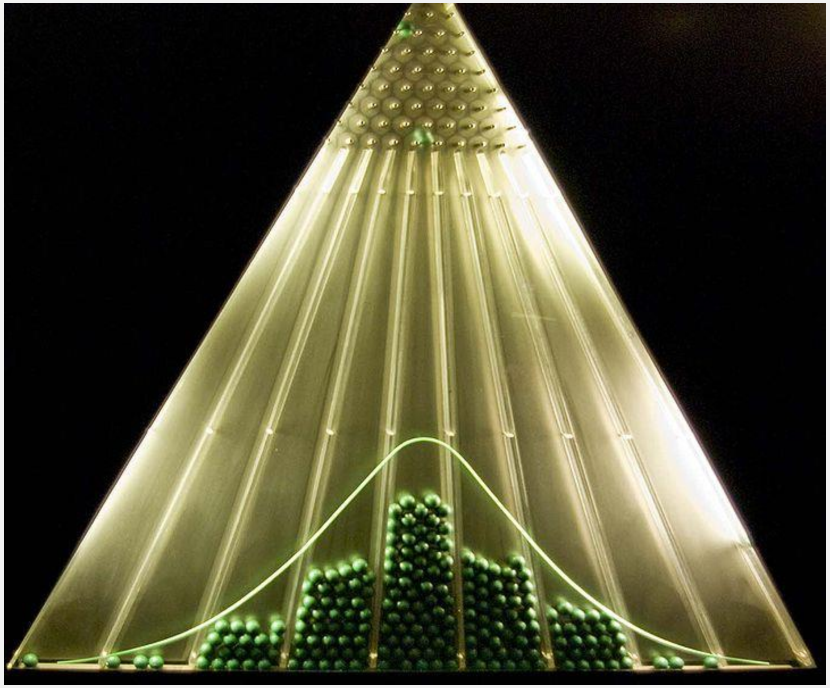
- Time consuming – slow performance
- Complicated – requires knowledge !!

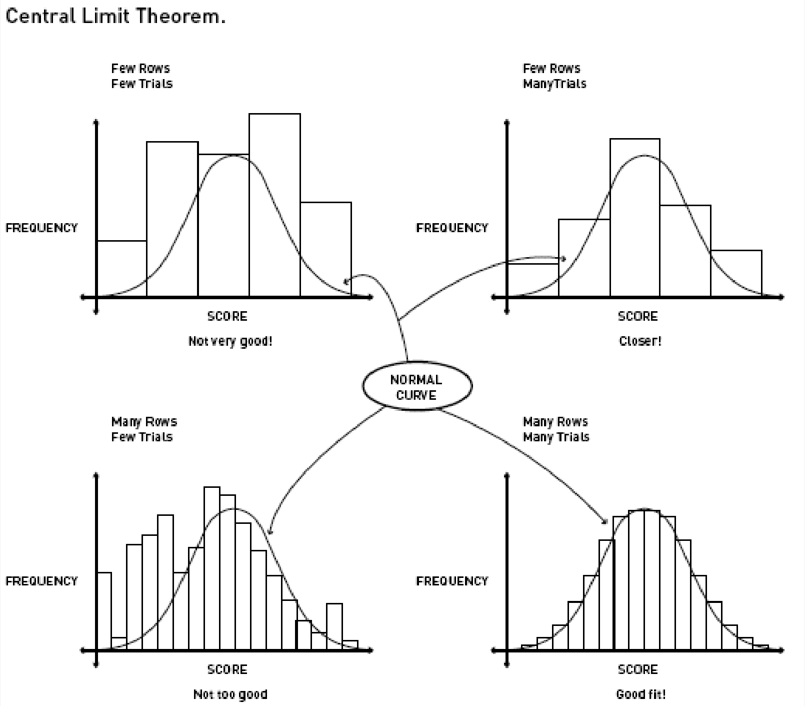
Data must have Normal – Gaussian - Distribution

GeoScene3D - Interpolation

Gauss Distribution and Central Limit Theorem

Many physical phenomena in nature arise from a number of additive variations. The distributions of the individual variations are often unknown, however, the histogram of the summed variable is often observed to be approximately Normal.





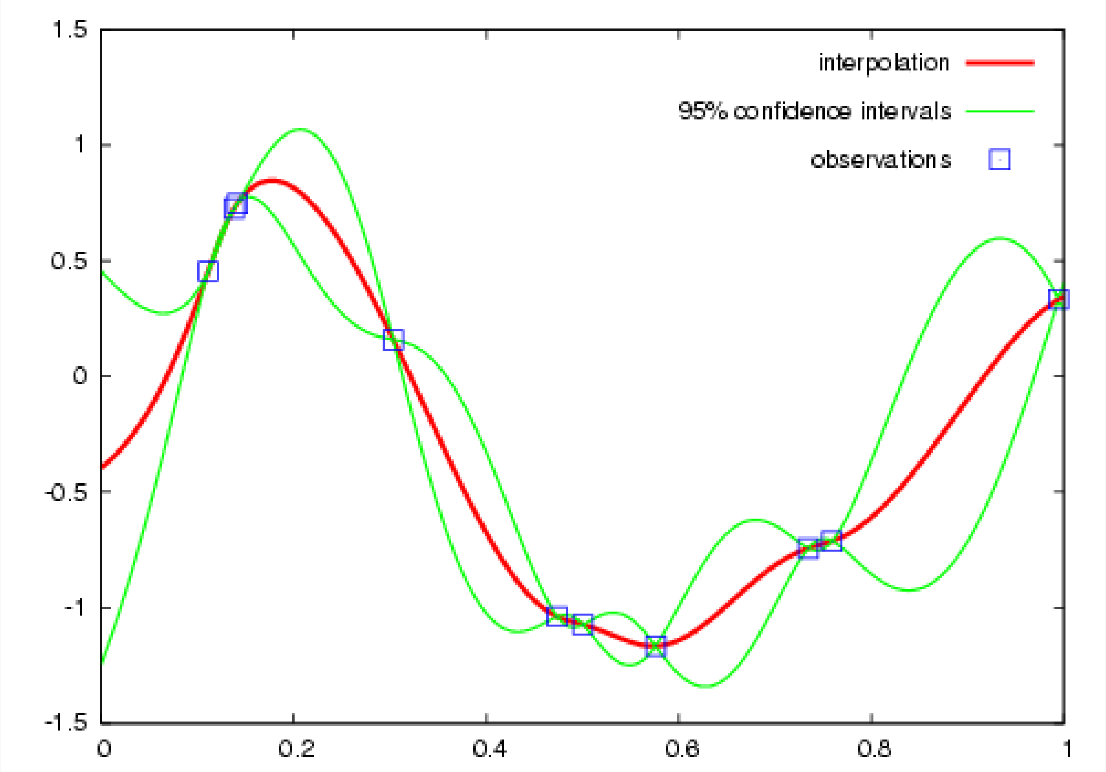
SAMPLING FREQUENCY !!

Example

Bean Machine – beans are dropped and randomly falls into binns. As by magic the Gaussian distribution appears ☺

GeoScene3D - Interpolation

Kriging
Interpolation based on an assumption that the data describes the sampling of a normally (Gaussian) distributed phenomena.

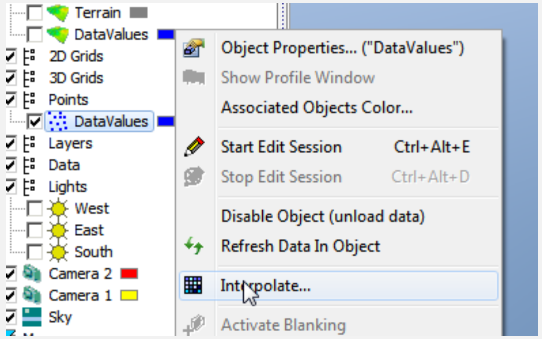


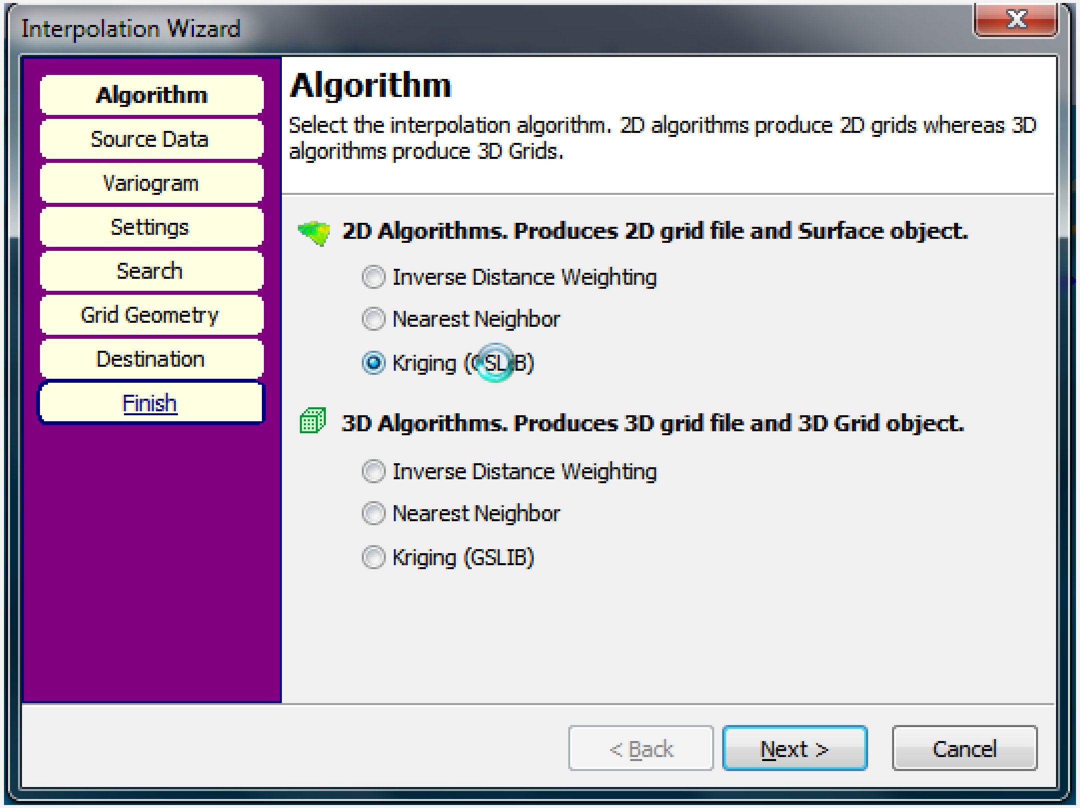
We get:
- A value estimate
- An uncertainty estimate

All based on the assumption of a GAUSSIAN distribution of our data !!!

Kriging example in 1 Dimension
Example of one-dimensional data interpolation by kriging, with confidence intervals. Squares indicate the location of the data. The kriging interpolation is in red. The confidence intervals are in green

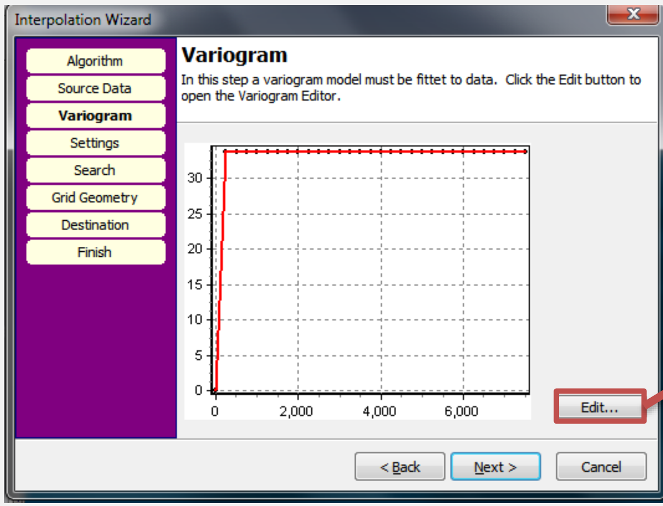
Kriging User Inter face and methodology in GeoScene3D



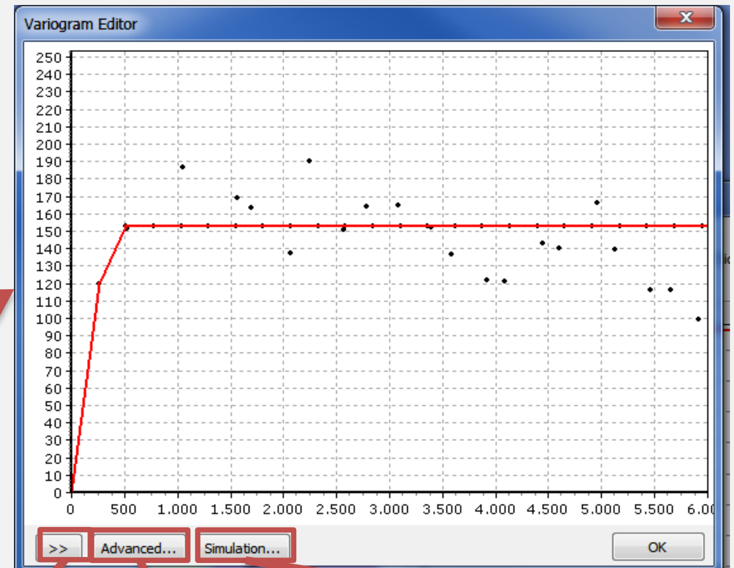


GeoScene3D - Interpolation

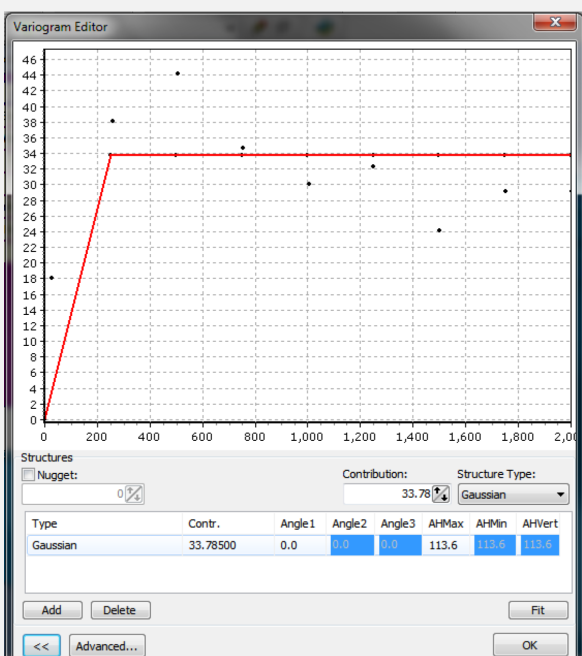
Variogram



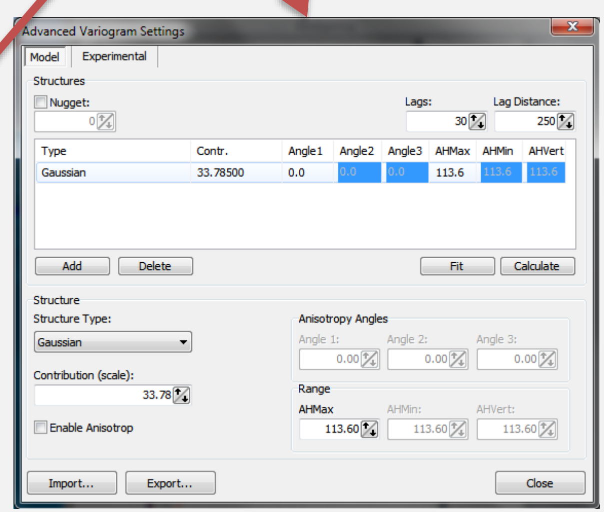
Variogram Editor - collapsed



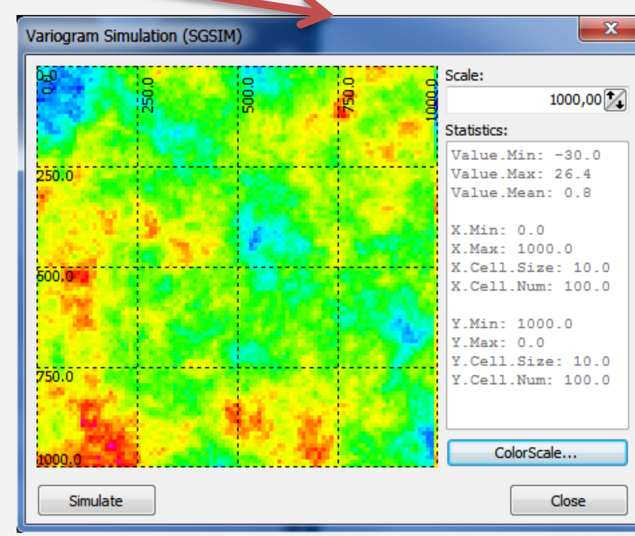
Expanded



Advanced

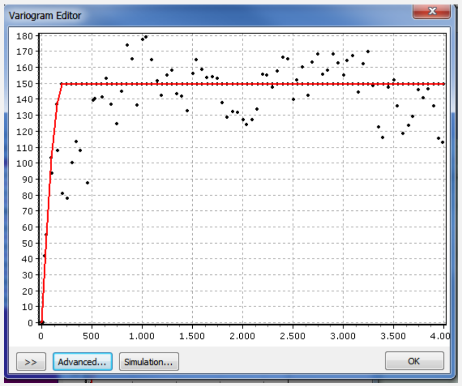
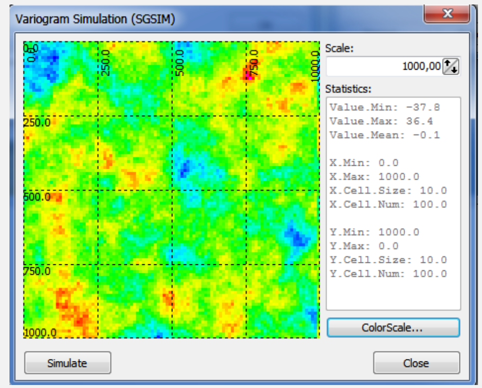
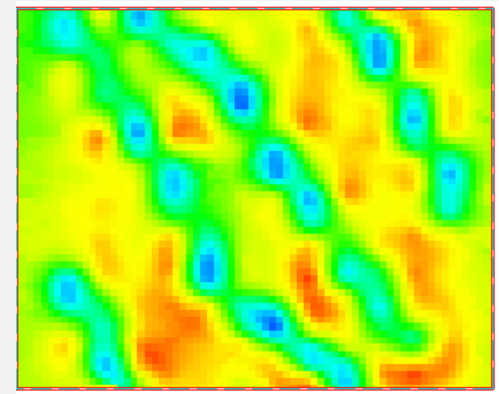
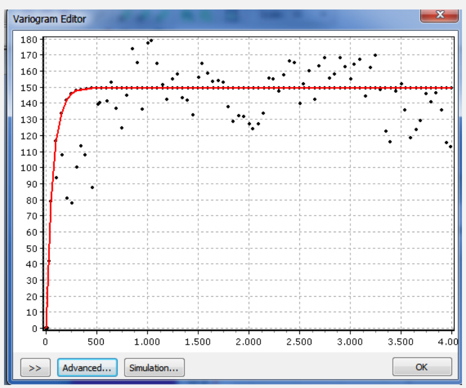
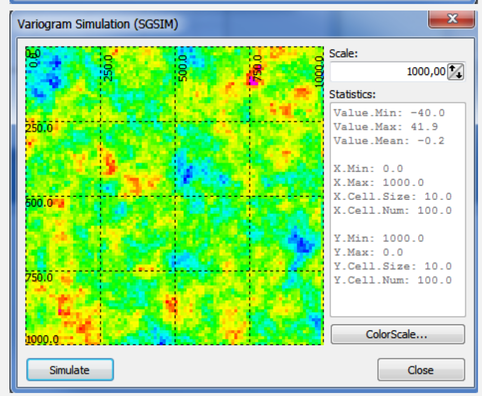
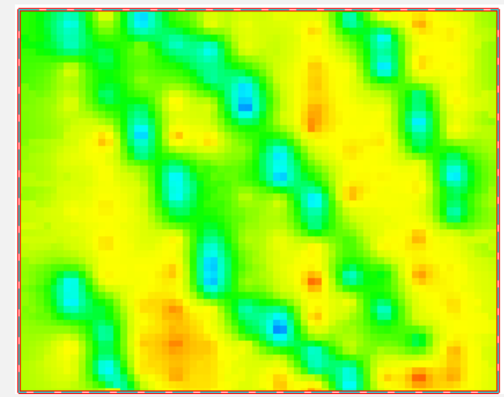
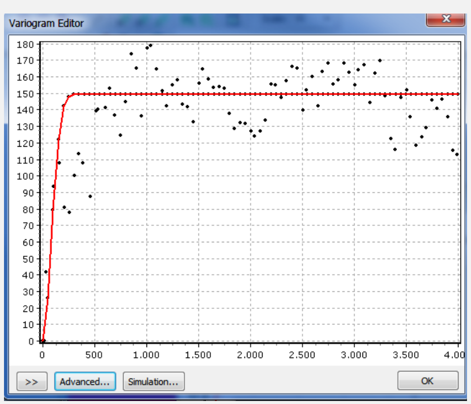
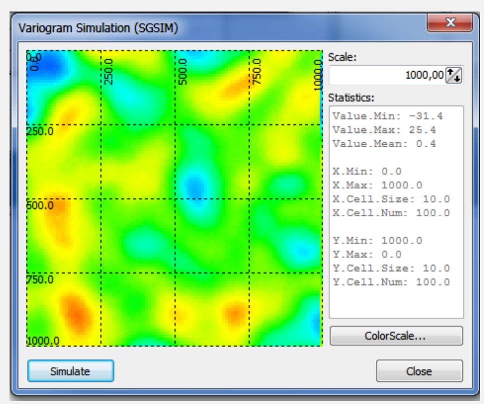
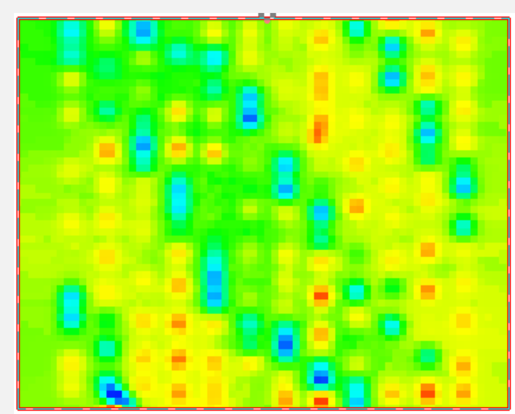


Simulation



GeoScene3D - Interpolation

GEOScene3D ...by I•GIS

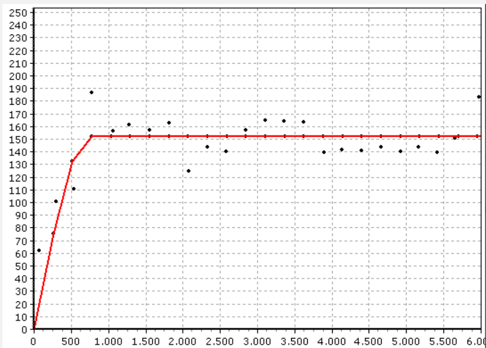
	Variogram	Simulation	Kriging Result
Spherical			
Exponential			
***Gaussian**			

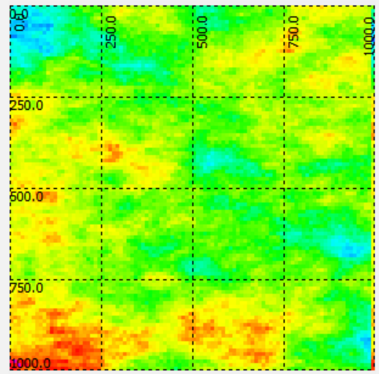
**NOTE: a nugget of 0.01 added!*

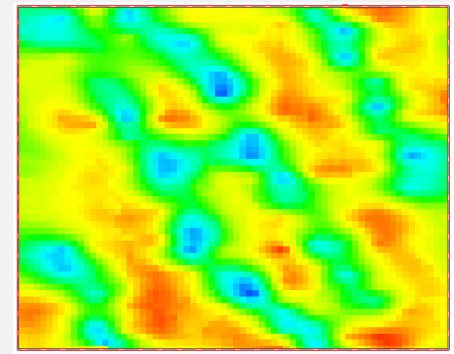
GeoScene3D - Interpolation

Variogram | **Simulation** | **Kriging Result**

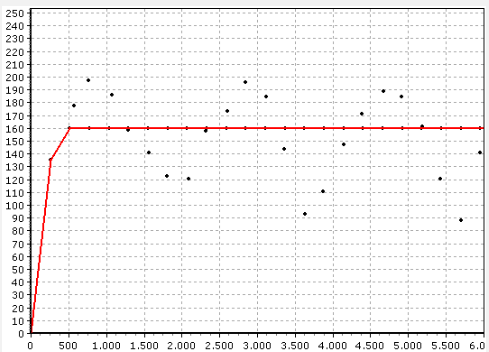
AZ=0

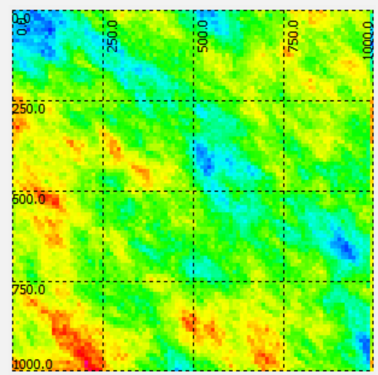


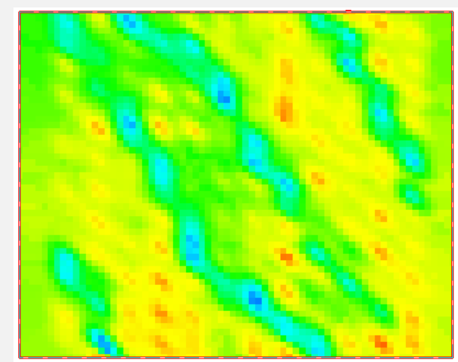




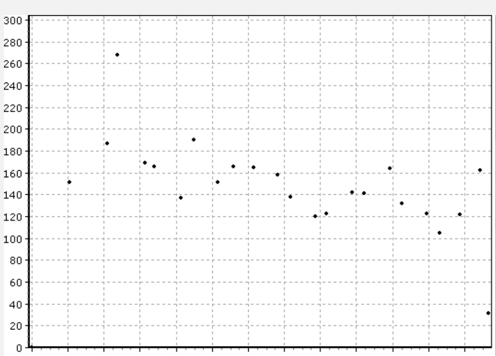
AZ=45

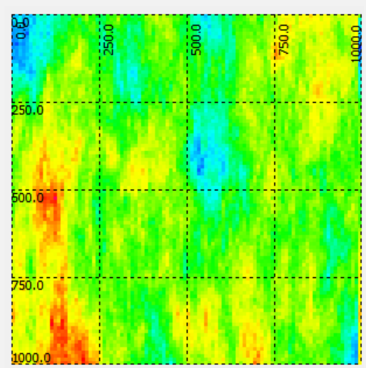


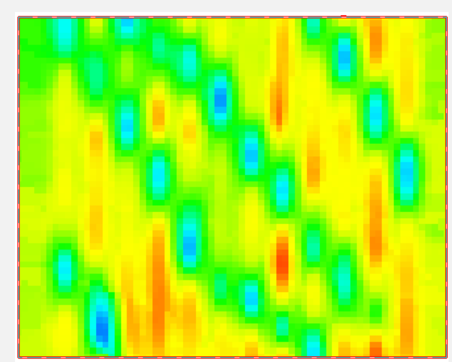




AZ=90



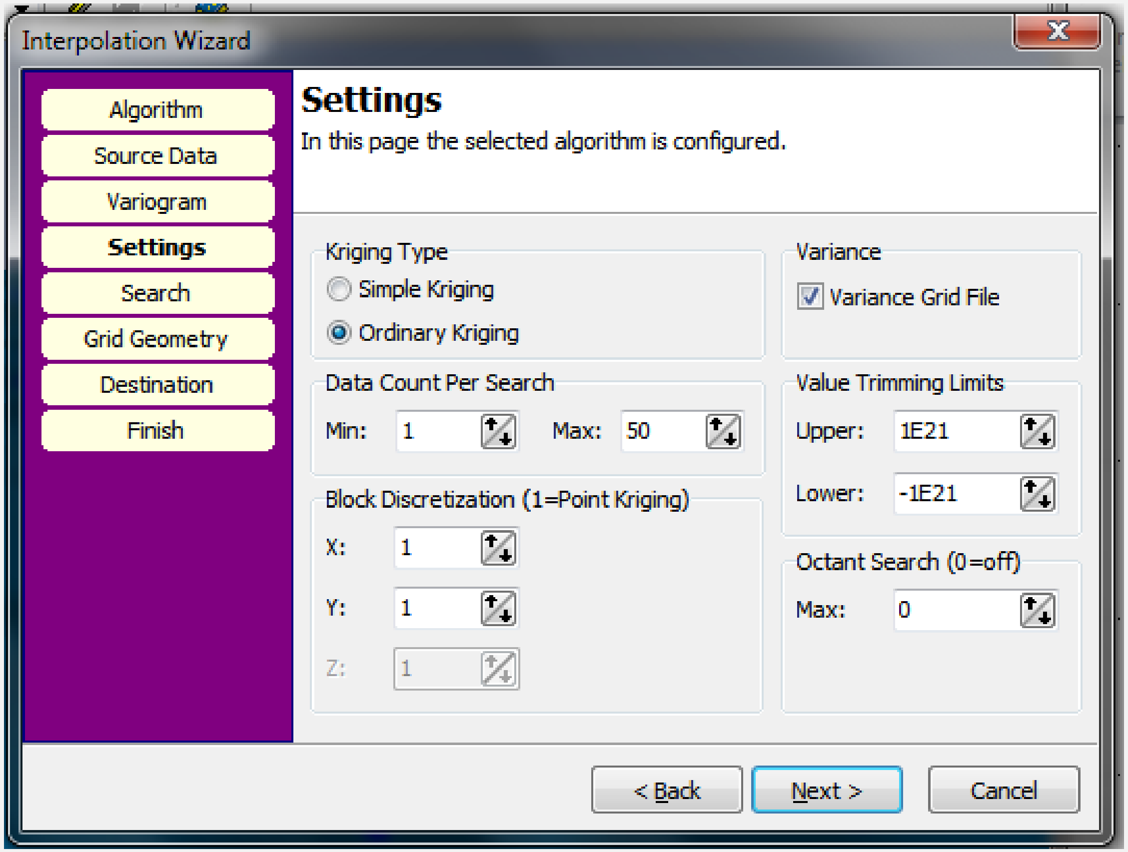




**NOTE: a nugget of 0.01 added!*

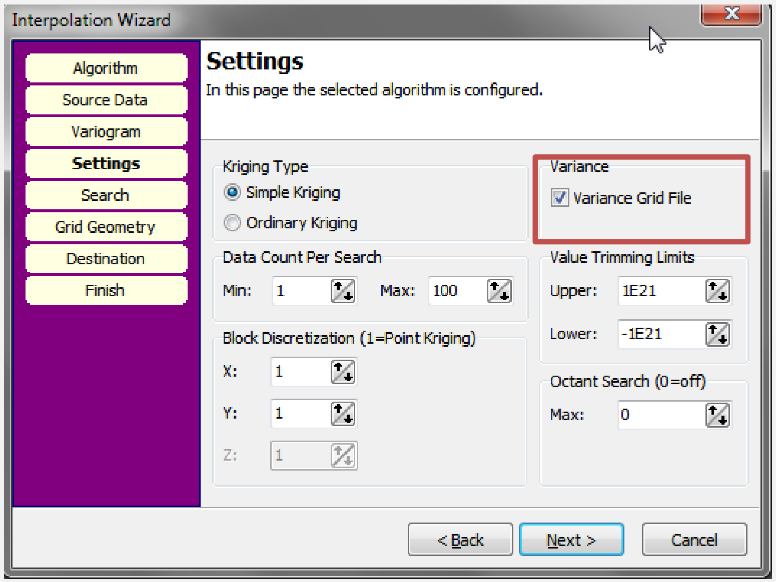
GeoScene3D - Interpolation

Settings



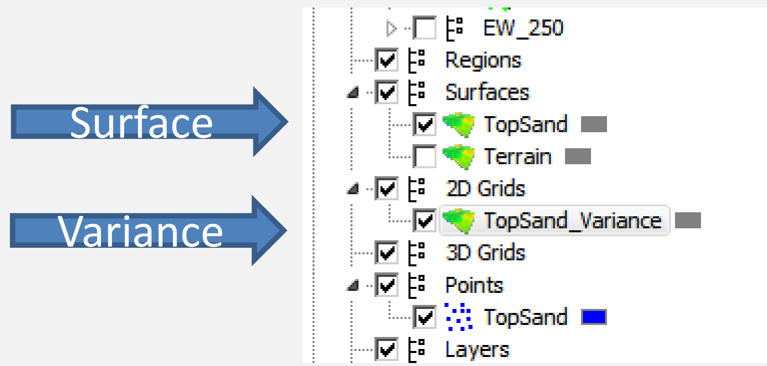
GEOScene3D
...by I•GIS

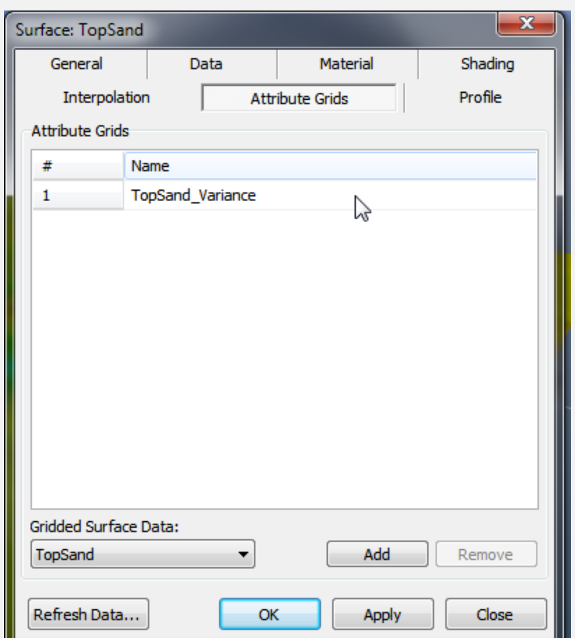
GeoScene3D - Interpolation

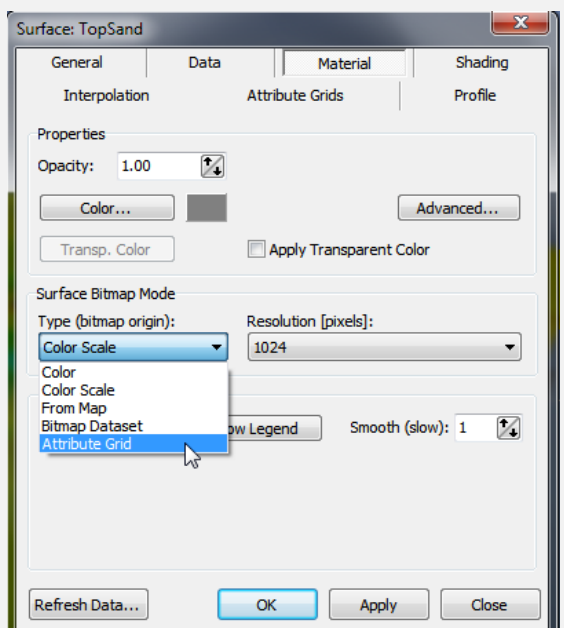


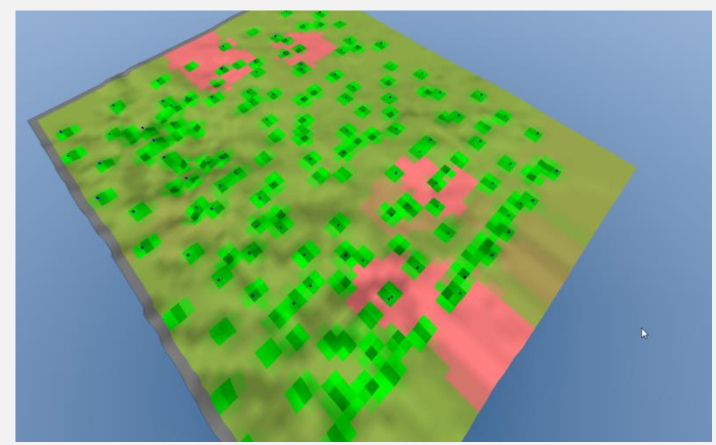
Variance Grid:

Check to produce a Variance Grid for the result.





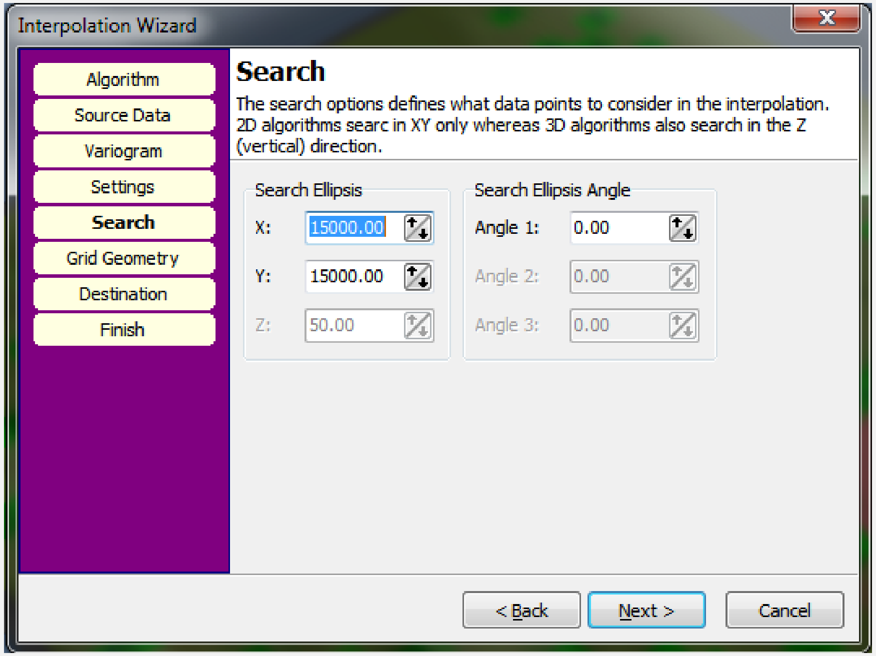


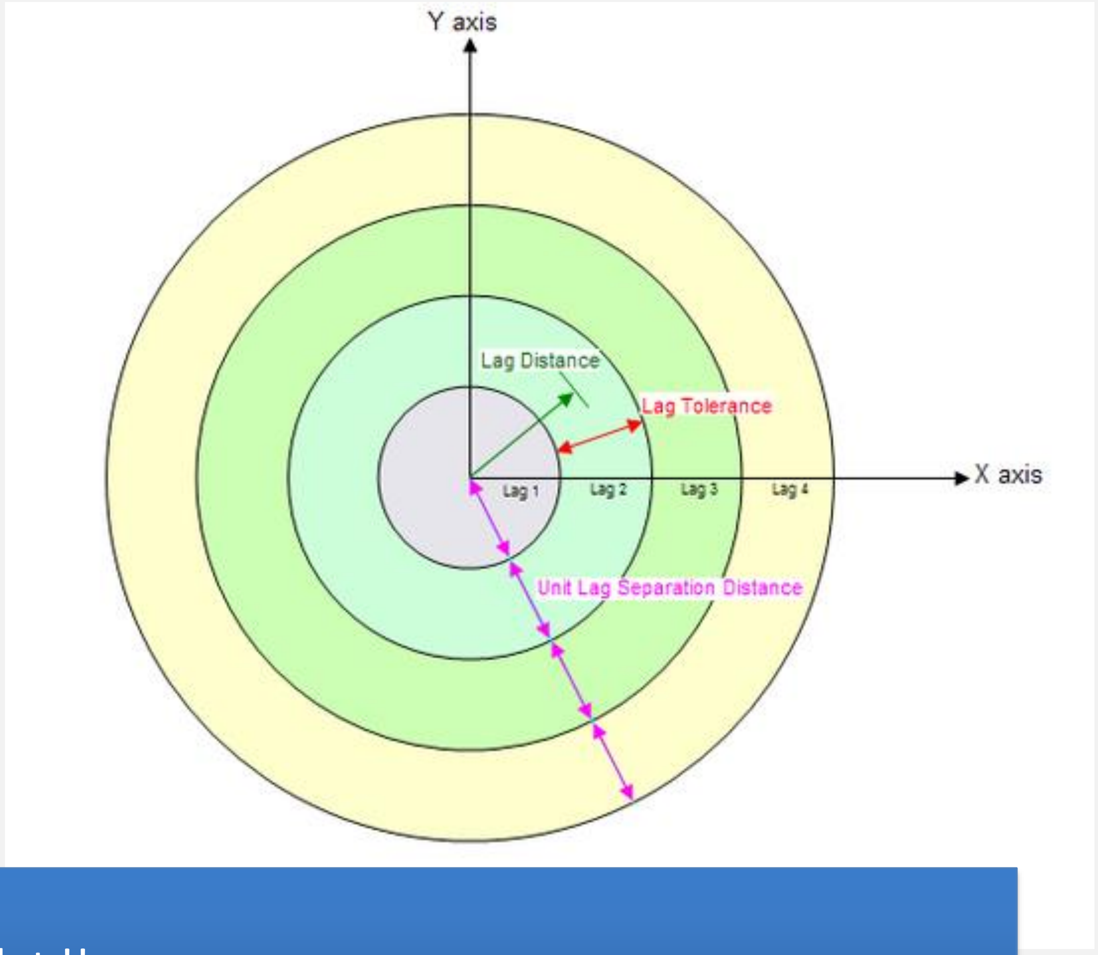


- Use it to check result
- Distribution of points
- Uncertainty analysis

GeoScene3D - Interpolation

Search

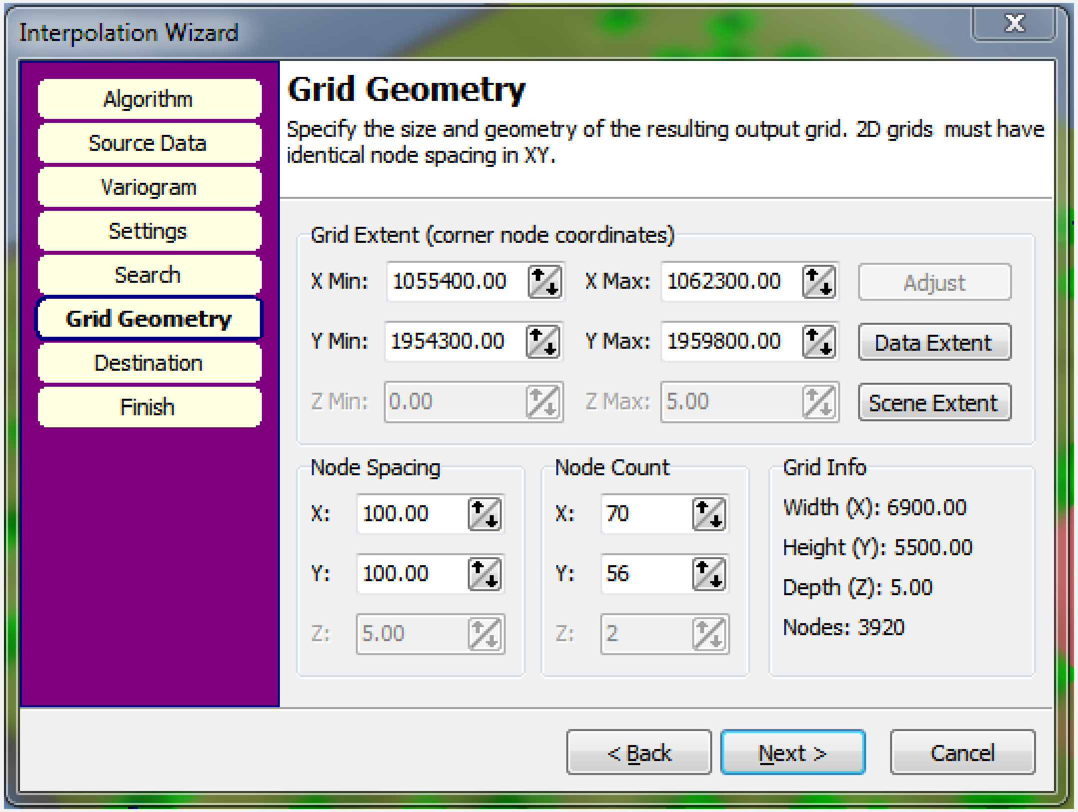


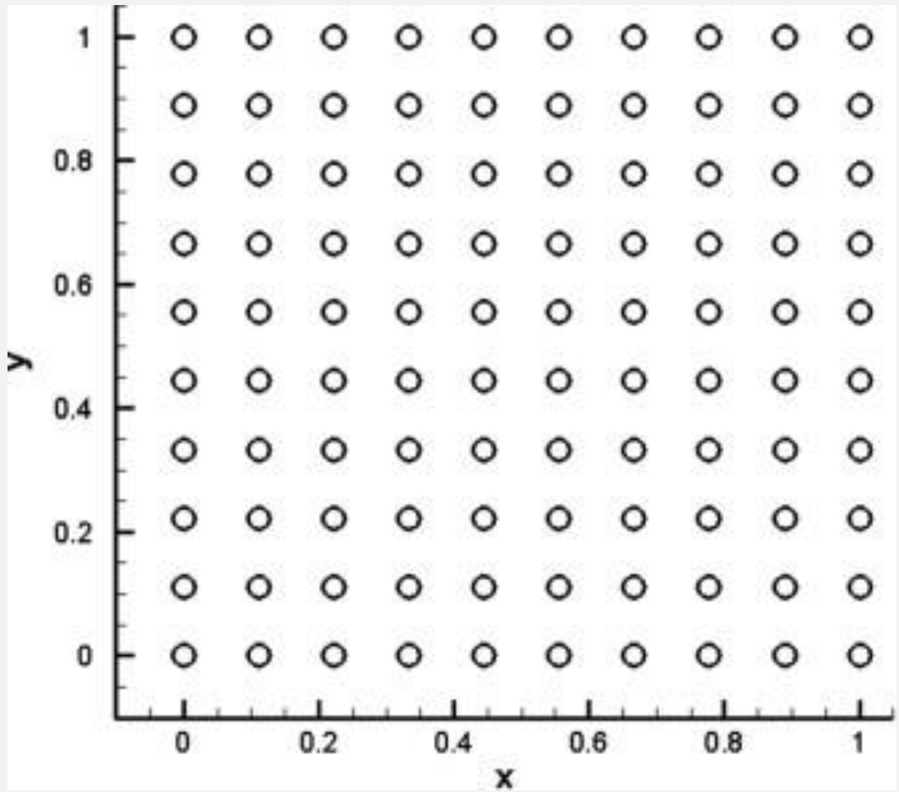


Define the Search Ellipses

- Remember distribution of data in Scatter plot !!
- Use “Data Count Per Search” to restrict to many points
- If using an Angle – experiment first with the experimental variogram to check for anisotropi !

Search

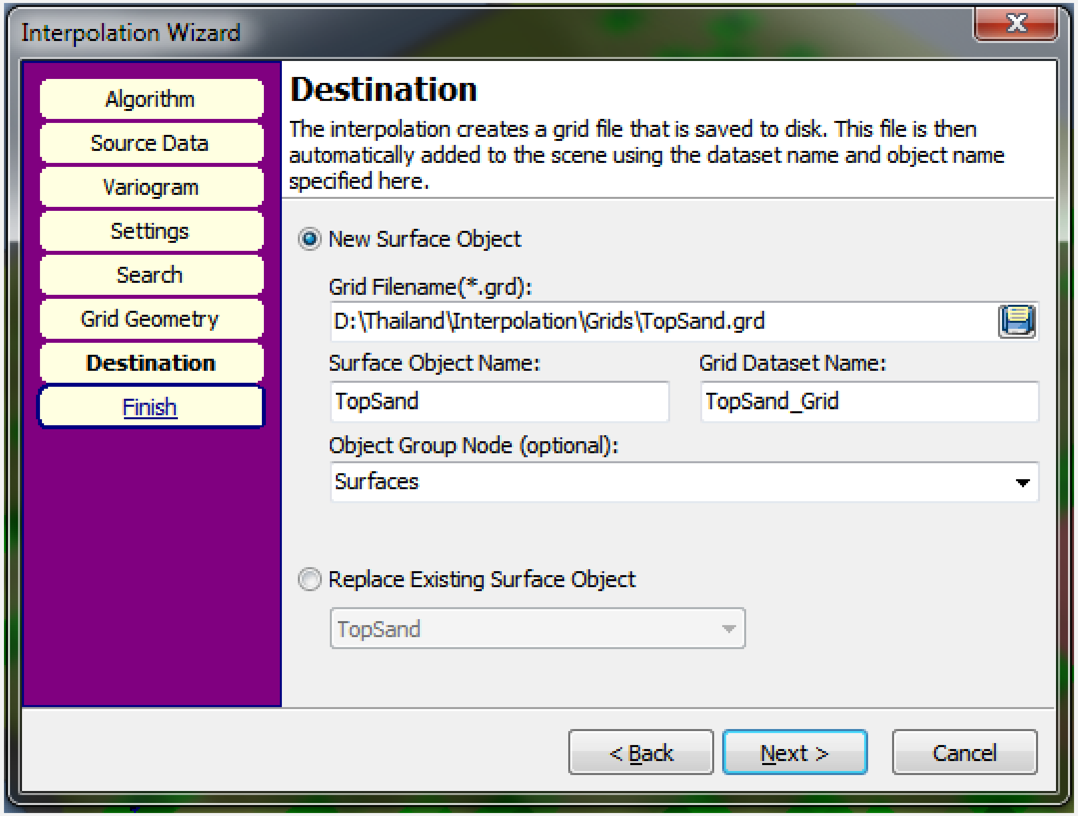




Define Resulting Grid

- Again - remember distribution of data in Scatter plot !!
- How detailed features do you wish (or are able) to resolve ?
- WARNING: Small grid cells potentially leads to very large grids !
- Be conservative!

GeoScene3D - Interpolation



General Rules of Thumb

- Check the data distribution – is it Gaussian?
- Check the spatial distribution of data
- Use the simplest model to fit the variogram
- If “islands” appears in you results, check the range used in the modeling !!
- Check that SILL corresponds to the variance you are expecting

Be aware that your choices in building the variogram deeply impacts the final result

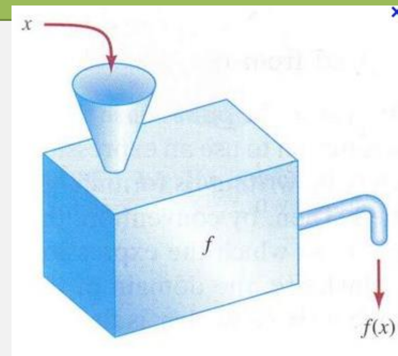
Many variables to consider – so be careful !

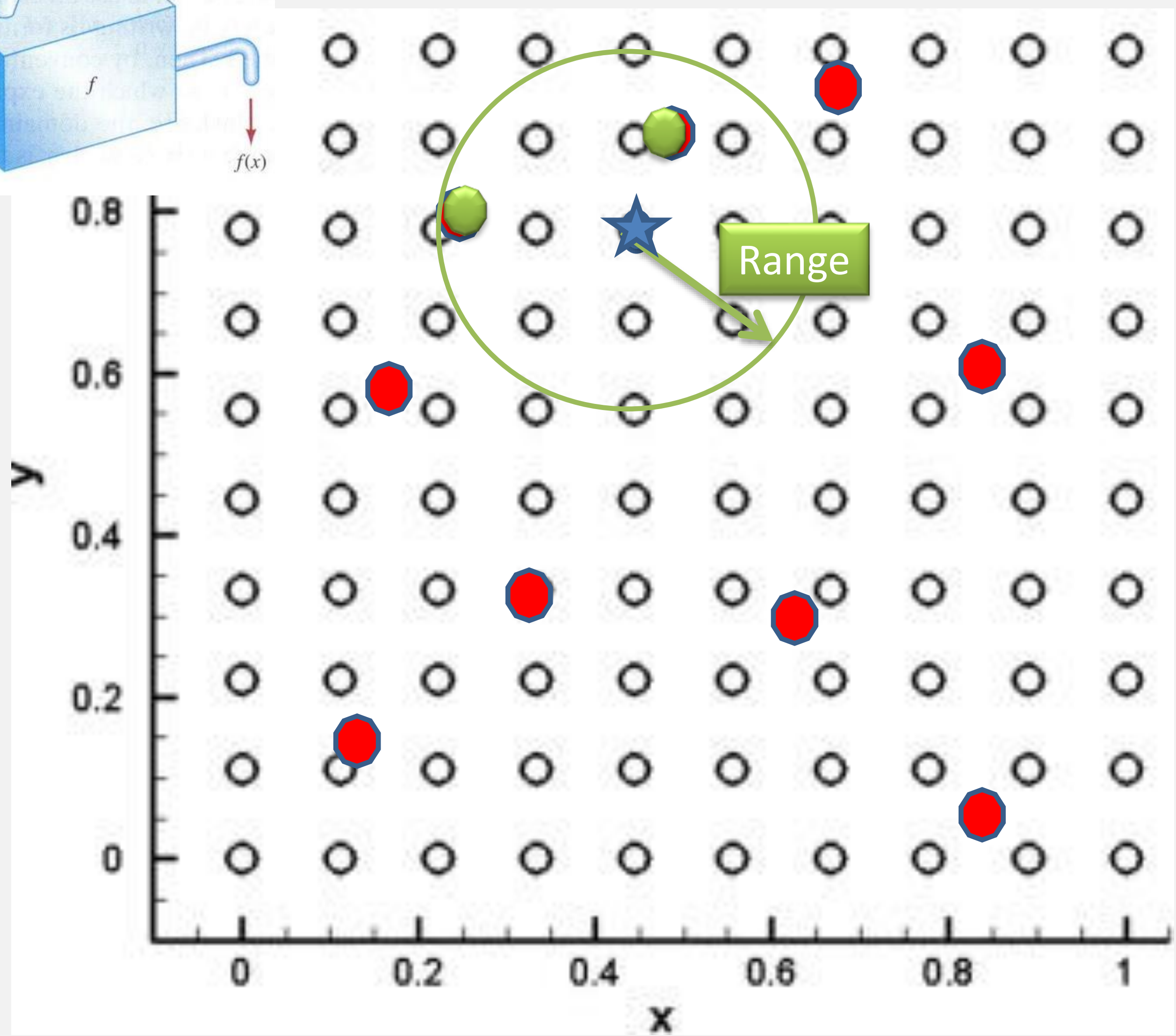
Kriging is a form of ART!

Agenda

- Interpolation defined
- General workflow in GeoScene3D
- Interpolation methods: Nearest-Neighbour (NN)
- Interpolation methods: Inverse Distance Weightning
- Interpolation methods: Kriging
- Summing up

GeoScene3D - Interpolation





We have our data

We have our empty grid

We estimate values in each node

Using a interpolation method

Which have a search range

Which we use to find the data to use in the interpolation.

And calculate an estimate In the cell node!

GeoScene3D - Interpolation

NN

- Simple – assign nearest value

IDW

- Provides smooth surfaces – assigns a value based on distance weight
- Removes trends in data
- Beware of range issues – can be handled using smoothing factor
- High Power=High Frequency; Low Power=Low Frequency

Kriging

- Statistical method – gives a value AND a measure of uncertainty
- Possibility for conserving trends in data (anisotropy)
- Variogram – a statistical tool for examining the data