

Basic Interpolation and Geostatistics

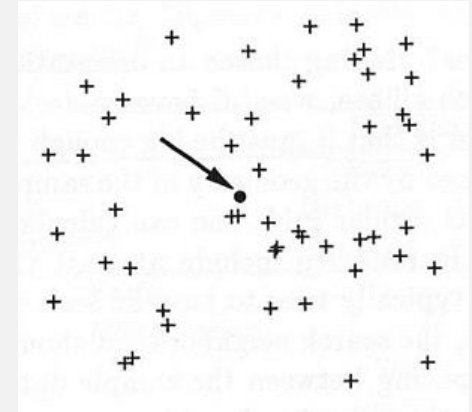
Interpolation in GeoScene3D

Agenda

- Interpolation defined
- General workflow in GeoScene3D
- Interpolation methods: Nearest-Neighbour (NN)
- Interpolation methods: Inverse Distance Weighted (IDW)
- Interpolation methods: Kriging
- Summing up

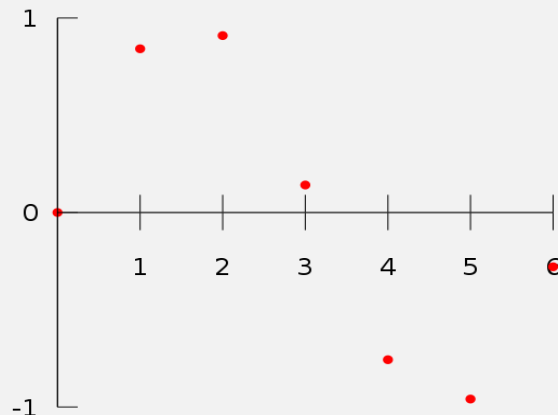
Interpolation

- A method of constructing new data points within the range of a discrete set of known data points.
- Interpolation is an estimation of values within known values from a sequence of values (surrounding known values)
- Irregular datapoints are interpolated to a regular grid



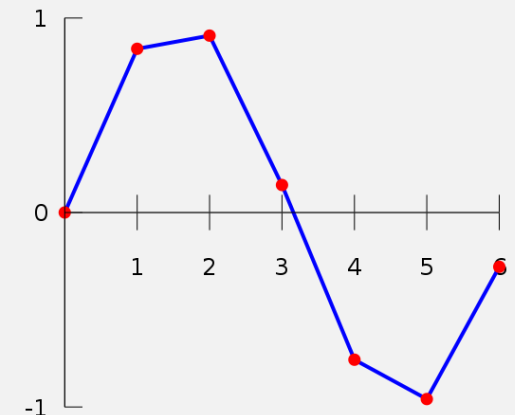
Known Data

x	$f(x)$
0	0
1	0.8415
2	0.9093
3	0.1411
4	-0.7568
5	-0.9589
6	-0.2794



Interpolation
Method

Interpolated Estimate

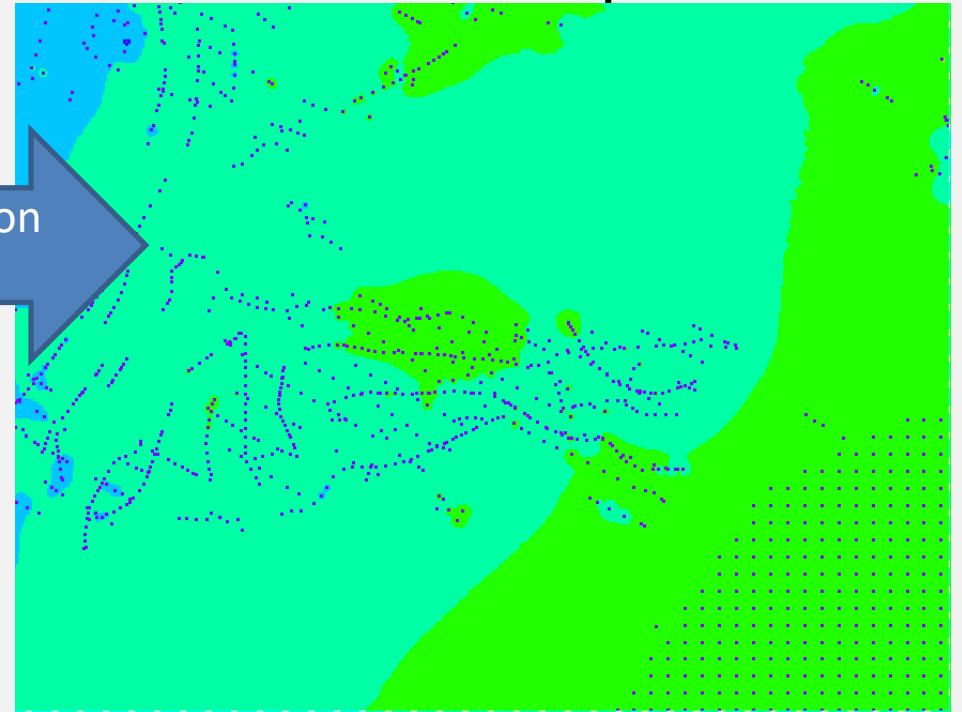


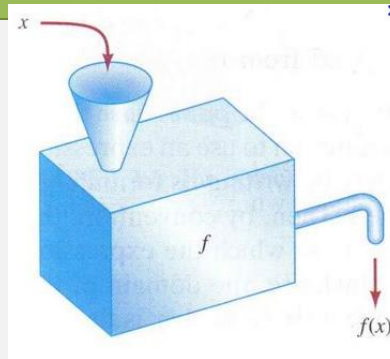
Known Data



Interpolation
Method

Interpolated Estimate





We have our *data*

We have our empty *grid*

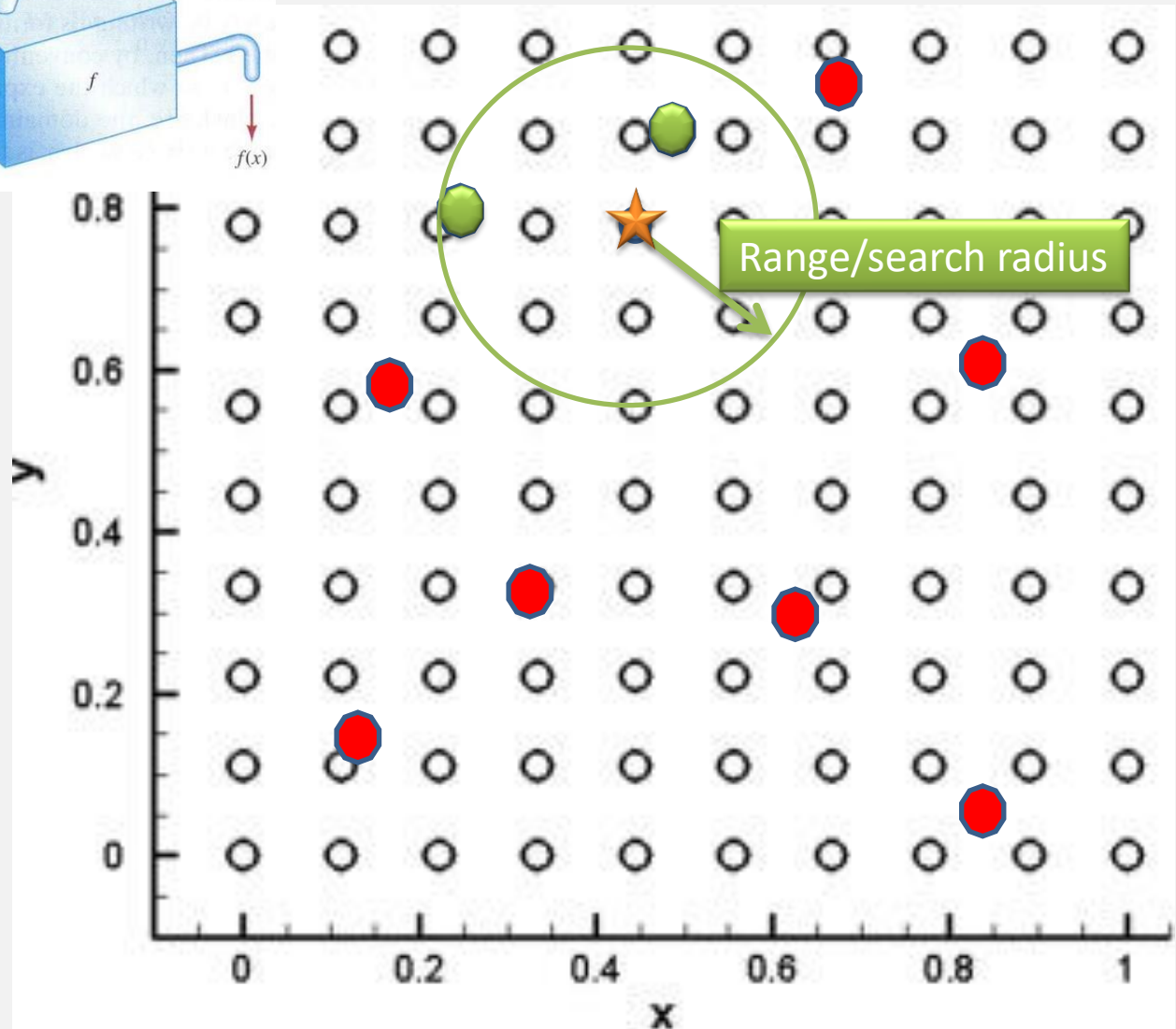
We estimate values in each *node*

Using an *interpolation method*

Within have a *search range*

Which we use to *define* the data to use in the interpolation.

And *calculate an estimate*
In the cell node!



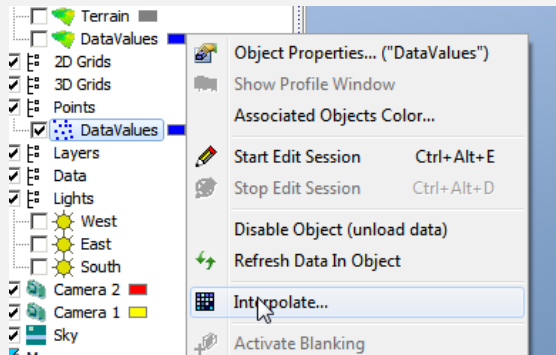
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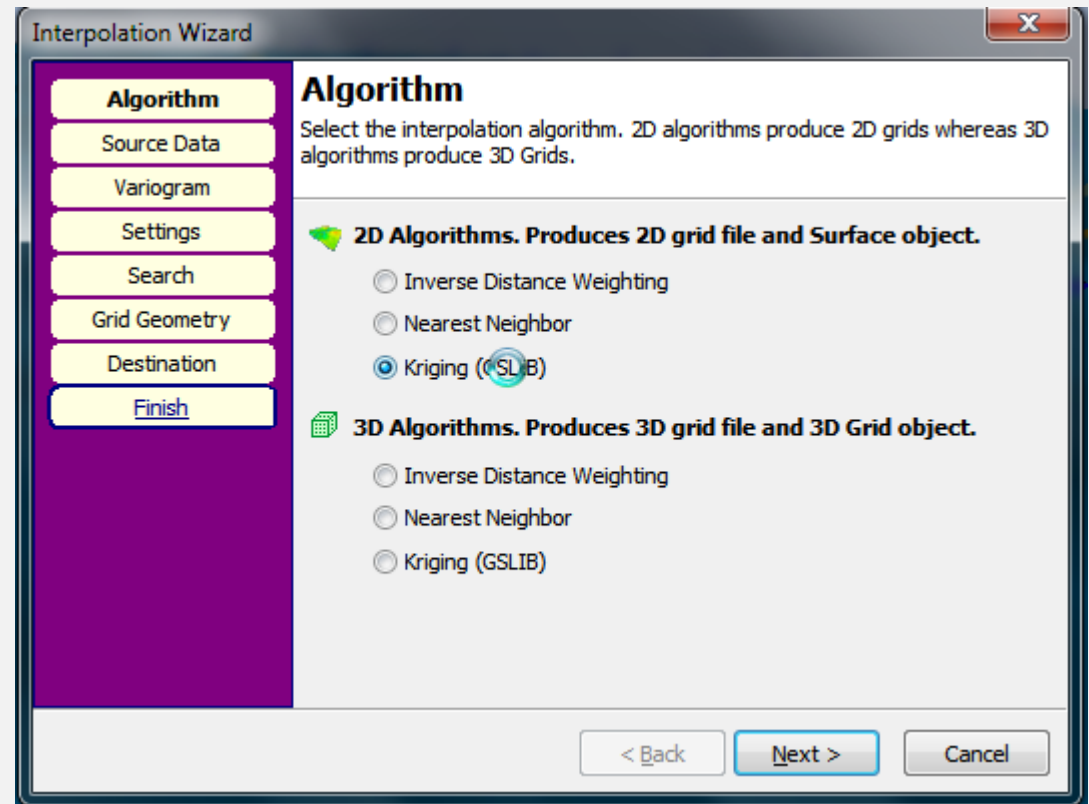
GeoScene3D - Interpolation

1. Choose you data for interpolation

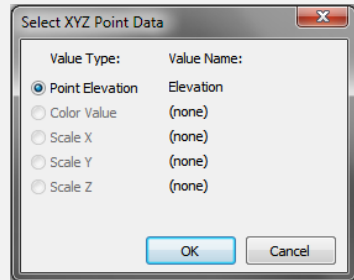
2. Choose interpolation algorithm



Activate

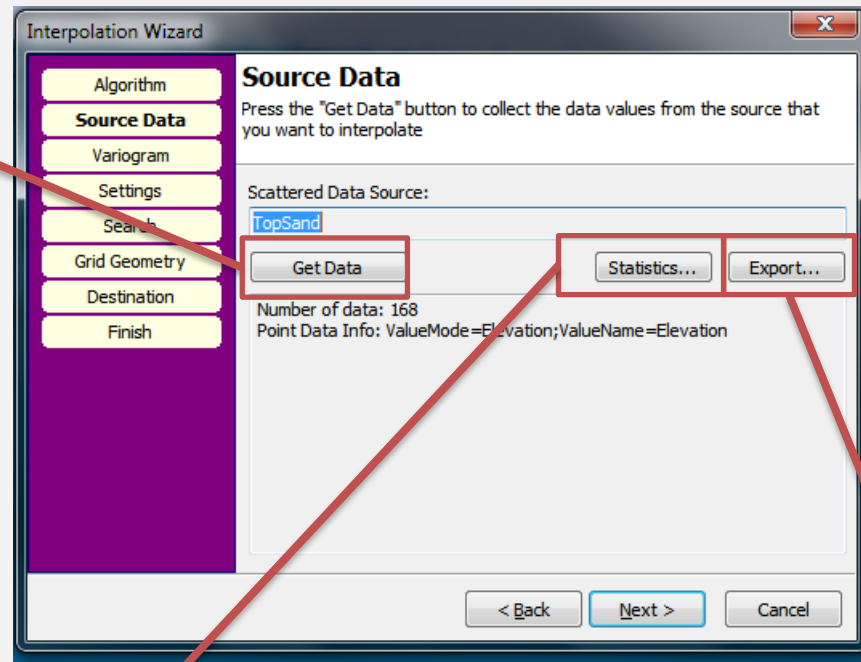


3: Select and inspect data...

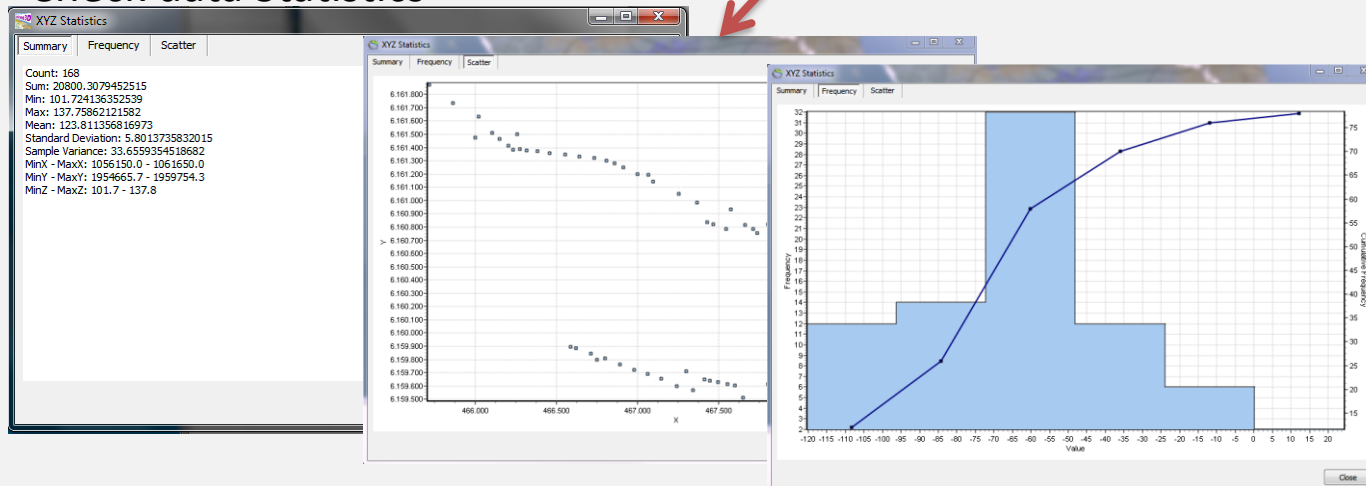


Select value to interpolate

Source Data Window



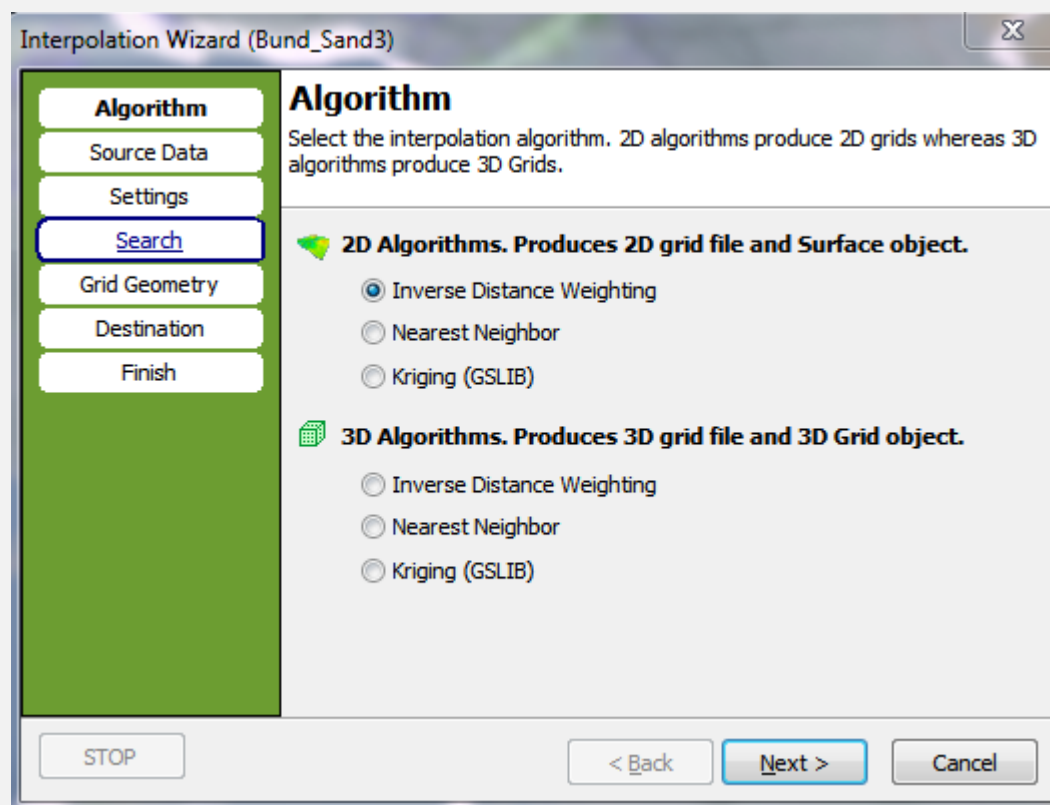
Check data Statistics



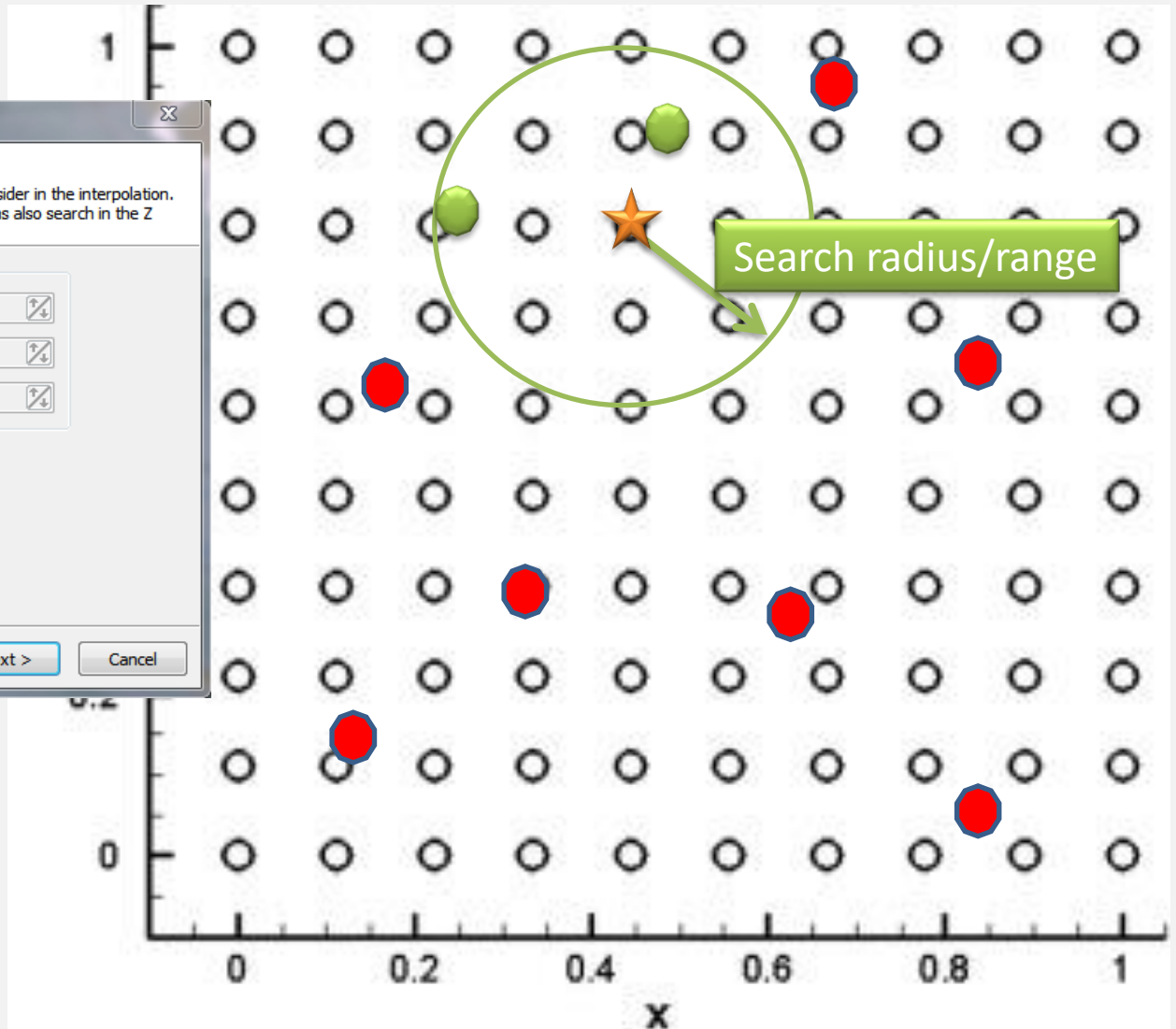
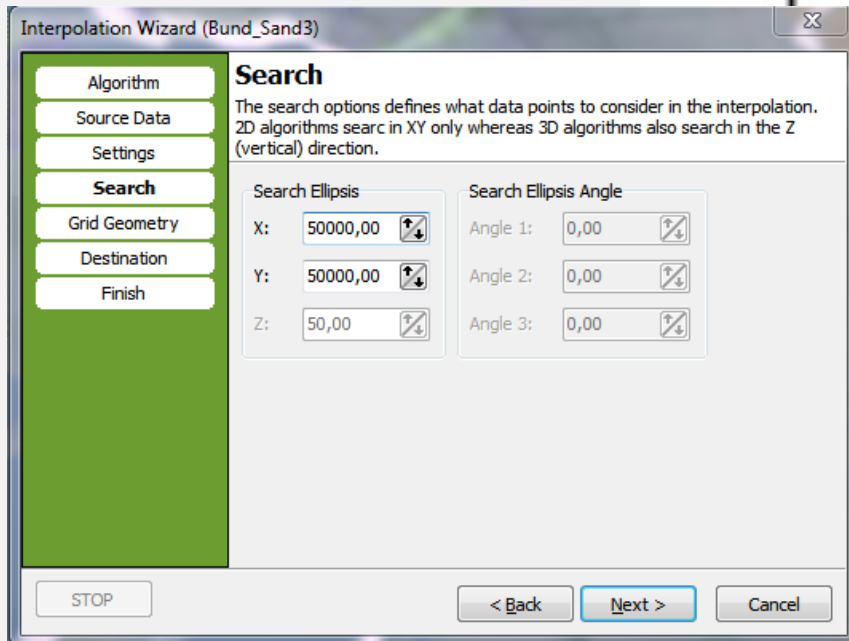
Export to CSV file

```
1 ID:X;Y;Z;Value
2 0;1056275.29052734;1958875;117.177917480469;117.177917480469
3 1;1056430.60717773;1958875;117.177917480469;117.177917480469
4 2;1057057.84716797;1958875;111.042945861816;111.042945861816
5 3;1057362.50671387;1958875;109.815956115723;109.815956115723
6 4;1057726.90332031;1958875;126.31901550293;126.31901550293
7 5;1058294.40649414;1958875;107.361961364746;107.361961364746
8 7;1059100.85791016;1958875;104.907974243164;104.907974243164
9 8;1059710.17675781;1958875;106.134971618652;106.134971618652
10 9;1060415.07568359;1958875;119.631896972656;119.631896972656
11 10;1060833.23535156;1958875;114.723922729492;114.723922729492
12 11;1061048.28955078;1958875;115.950912475586;115.950912475586
13 12;1056215.70678711;1958625;126.206893920898;126.206893920898
14 13;1056760.20947266;1958625;126.206893920898;126.206893920898
15 14;1057357.06811523;1958625;121.37931060791;121.37931060791
16 15;1058137.17285156;1958625;127.931037902832;127.931037902832
17 16;1058938.2199707;1958625;128.275863647461;128.275863647461
18 17;1059603.14160156;1958625;124.462757568359;124.462757568359
19 18;1060001.046875;1958625;103.448280334473;103.448280334473
20 19;1060461.78027344;1958625;128.275863647461;128.275863647461
21 20;1060765.44482422;1958625;129.655166625977;129.655166625977
22 21;1056407.75622559;1958375;126.551719665527;126.551719665527
23 22;1057363.43493652;1958375;124.827590942383;124.827590942383
24 23;1058374.51513672;1958375;123.103446960449;123.103446960449
25 24;1059136.28808594;1958375;129.655166625977;129.655166625977
```

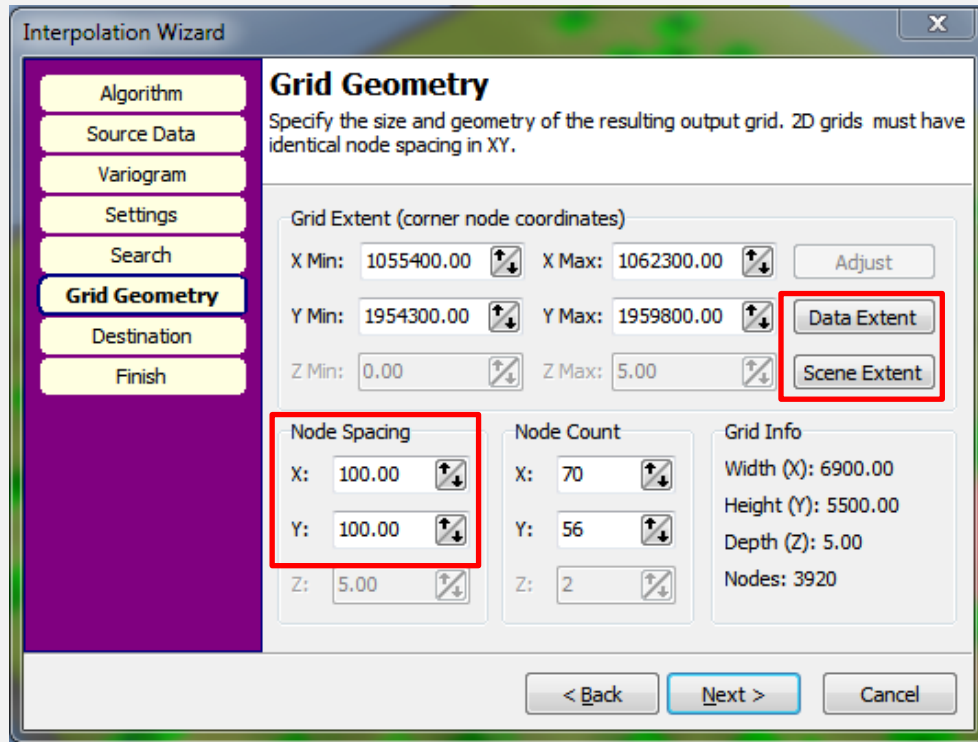

4: Specify settings for the algorithm selected...



5: Define Search Range...



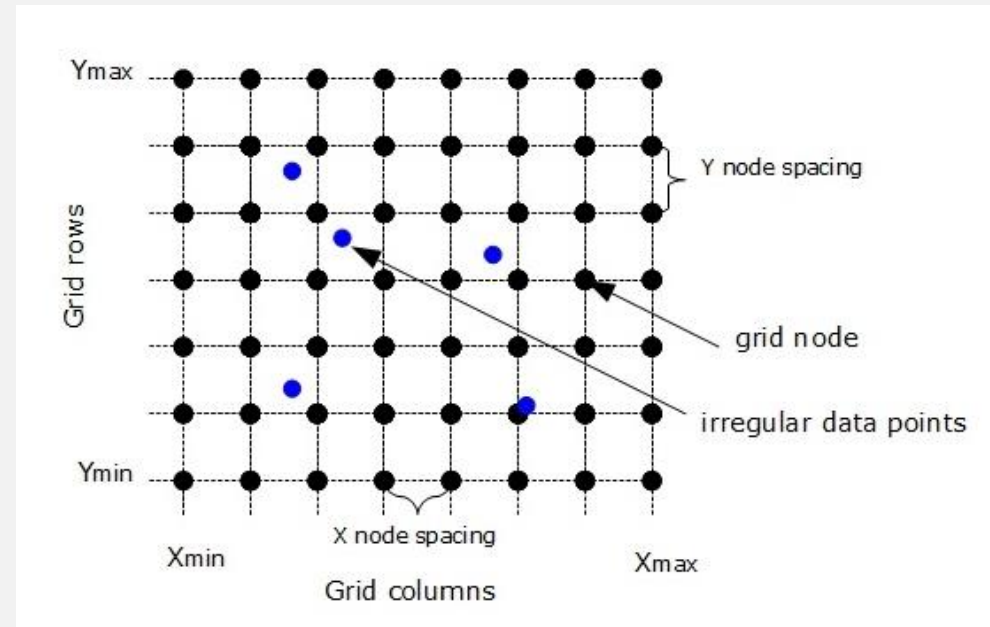
Grid Geometry and Data Extent



The screenshot shows the 'Interpolation Wizard' dialog box with the 'Grid Geometry' tab selected. The dialog is divided into a left sidebar with buttons for 'Algorithm', 'Source Data', 'Variogram', 'Settings', 'Search', 'Grid Geometry' (highlighted), 'Destination', and 'Finish'. The main area contains the following sections:

- Grid Geometry**: A title bar with a subtitle 'Specify the size and geometry of the resulting output grid. 2D grids must have identical node spacing in XY.'
- Grid Extent (corner node coordinates)**: Fields for X Min (1055400.00), X Max (1062300.00), Y Min (1954300.00), Y Max (1959800.00), Z Min (0.00), and Z Max (5.00). An 'Adjust' button is next to the X Max field. 'Data Extent' and 'Scene Extent' buttons are also present.
- Node Spacing**: Fields for X (100.00), Y (100.00), and Z (5.00).
- Node Count**: Fields for X (70), Y (56), and Z (2).
- Grid Info**: A summary box showing Width (X): 6900.00, Height (Y): 5500.00, Depth (Z): 5.00, and Nodes: 3920.

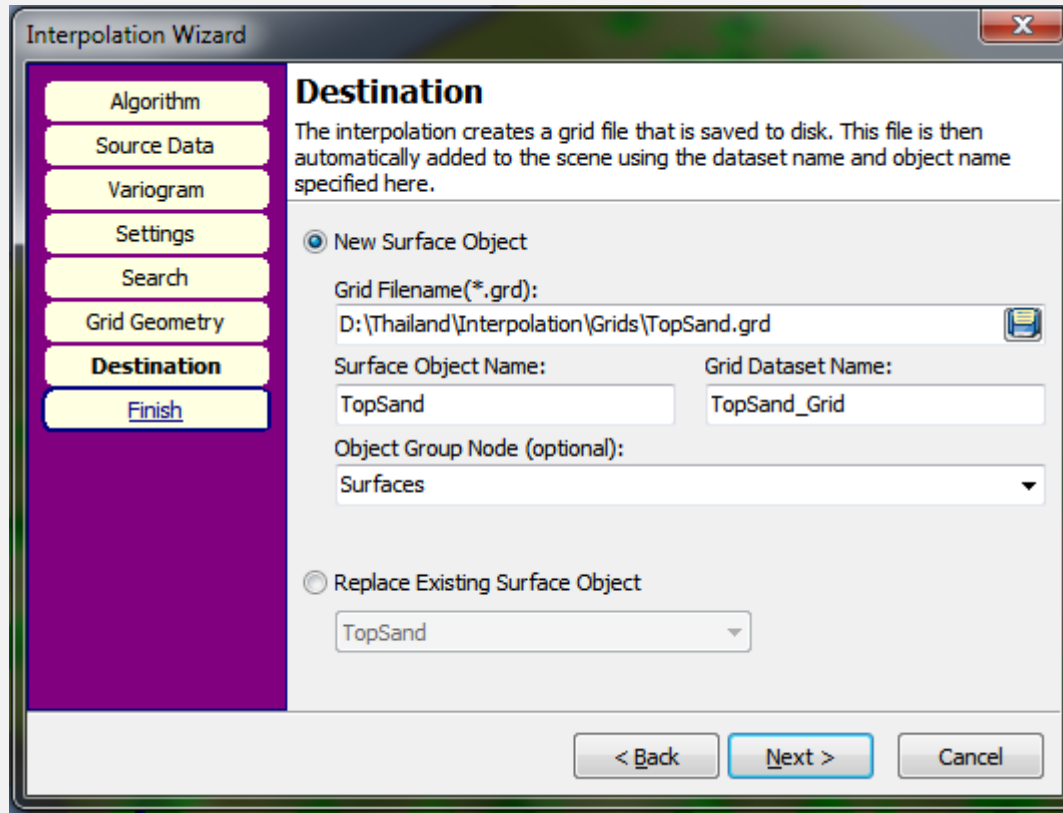
Navigation buttons at the bottom include '< Back', 'Next >', and 'Cancel'.



Define Resulting Grid

- How detailed features do you wish (or are able) to resolve ?
- WARNING: Small grid cells potentially leads to very large grids !
- Be conservative!

Destination



The screenshot shows the 'Interpolation Wizard' dialog box, specifically the 'Destination' step. On the left is a vertical sidebar with buttons for 'Algorithm', 'Source Data', 'Variogram', 'Settings', 'Search', 'Grid Geometry', 'Destination' (which is highlighted), and 'Finish'. The main area has a title 'Destination' and a description: 'The interpolation creates a grid file that is saved to disk. This file is then automatically added to the scene using the dataset name and object name specified here.' There are two radio button options: 'New Surface Object' (selected) and 'Replace Existing Surface Object'. Under 'New Surface Object', there are fields for 'Grid Filename(*.grd):' with the value 'D:\Thailand\Interpolation\Grids\TopSand.grd', 'Surface Object Name:' with 'TopSand', 'Grid Dataset Name:' with 'TopSand_Grid', and 'Object Group Node (optional):' with a dropdown menu showing 'Surfaces'. Under 'Replace Existing Surface Object', there is a dropdown menu showing 'TopSand'. At the bottom are buttons for '< Back', 'Next >', and 'Cancel'.

- Save in a project folder
- Be consistent !
- Avoid clutter in your project structure !



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Nearest Neighbour

Description

At each grid point a value is assigned the value of the nearest data point within the search radius.

Parameters

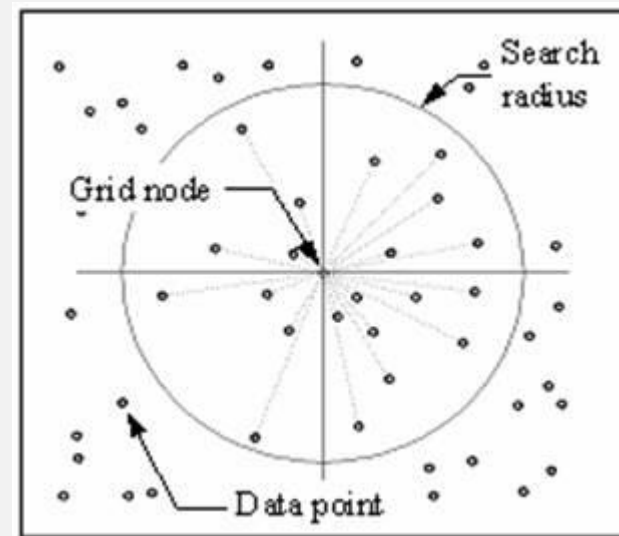
- None

Advantages

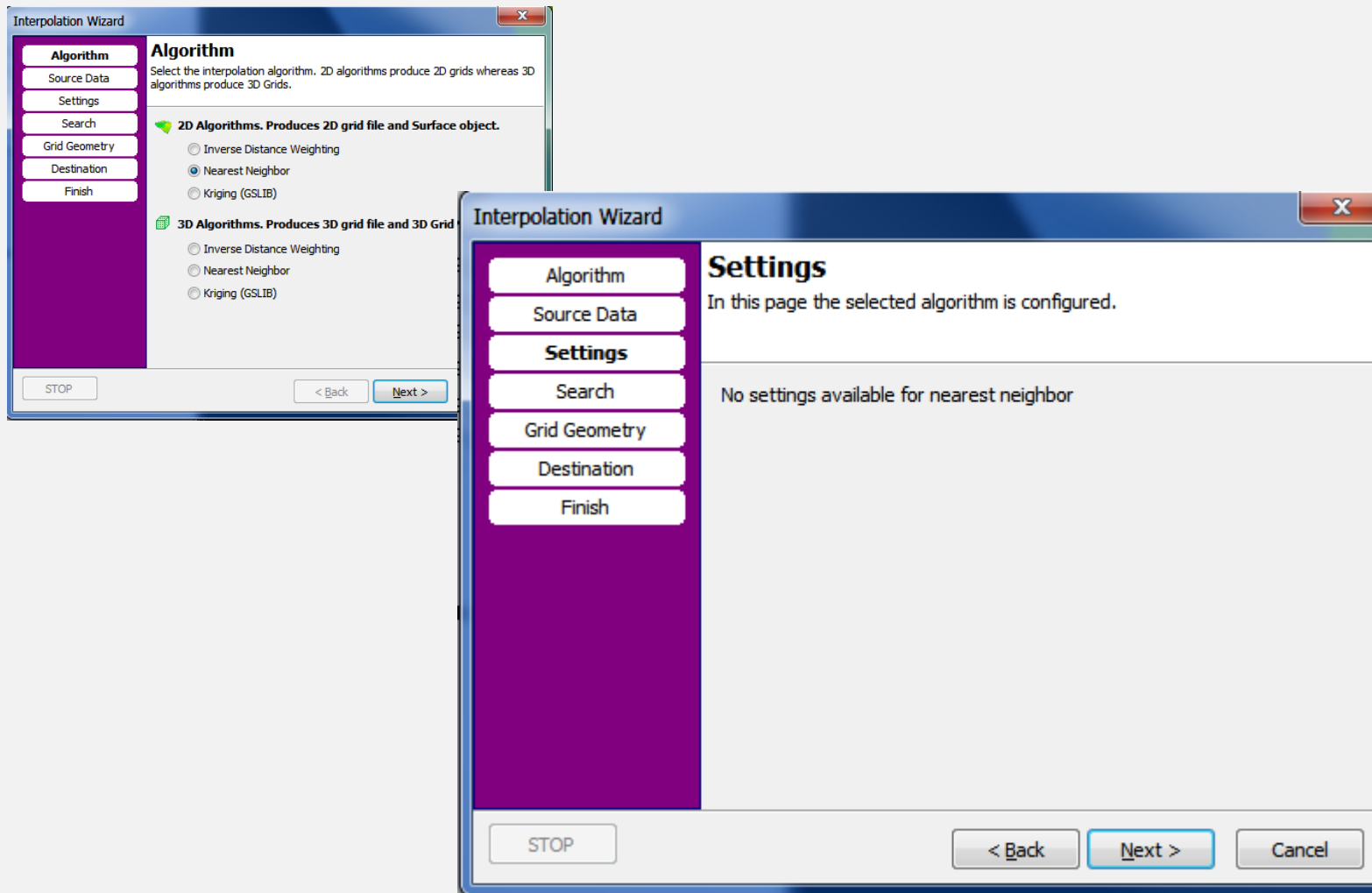
- Simple and fast
- Keeps high and low extreme values in the interpolated surfaces.

Disadvantages

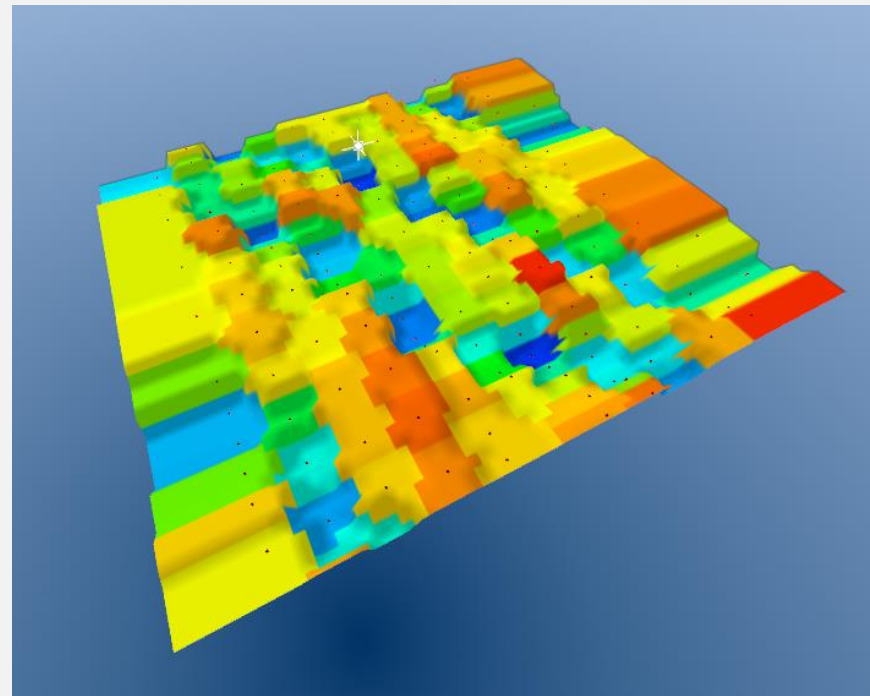
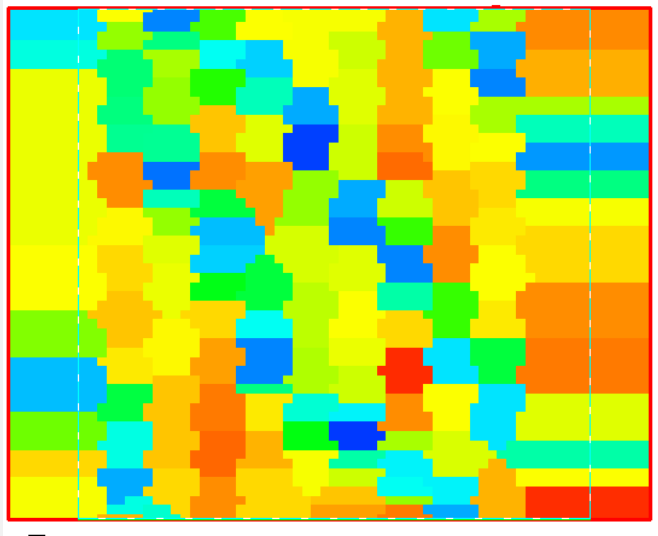
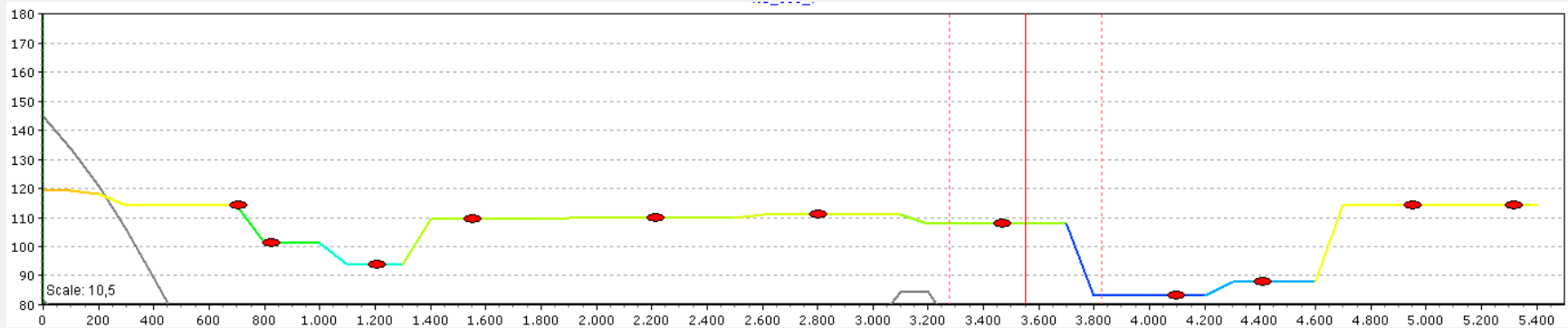
- Maybe to simple 😊
- Results in high frequency “noisy” surface



Nearest Neighbour in GeoScene3D



GeoScene3D - Interpolation



When would this be an appropriate way to interpolate data?

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Inverse Distance Weighted (IDW)

Description

At each grid point a value is assigned based on a *weighted mean* of the values within the search radius. The weighting is done by distance lifted to a Power.

Parameters

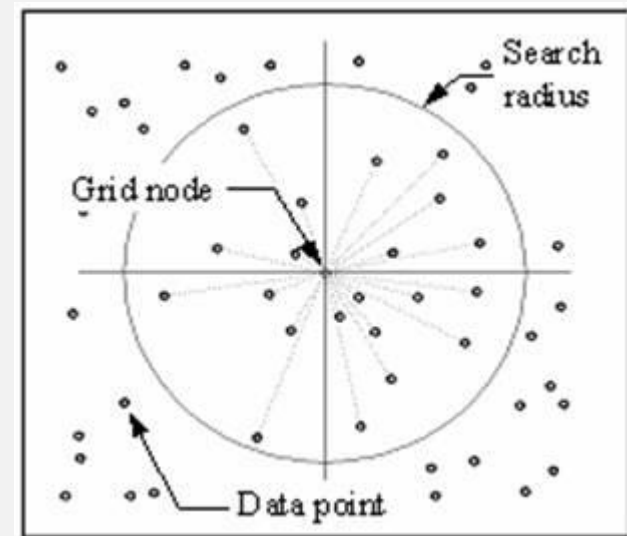
- Search Radius
- Exponent (Power)
- Smooth factor
- Search direction
- Point Limits / Quadrant Search

Advantages

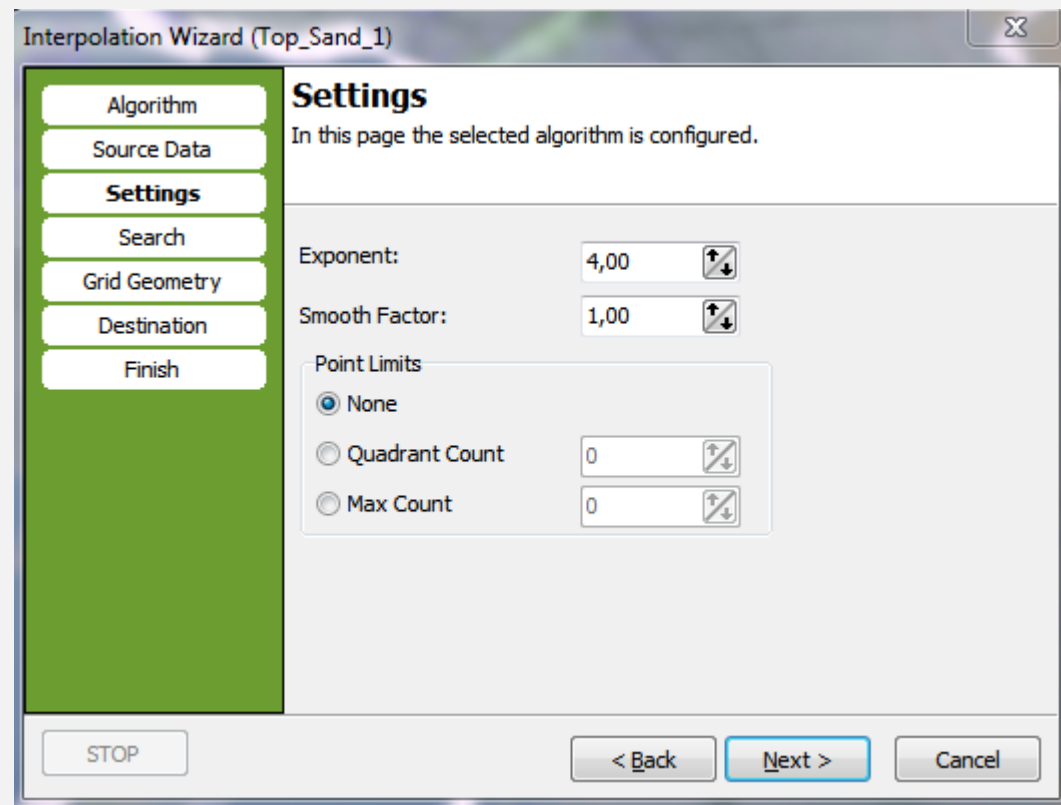
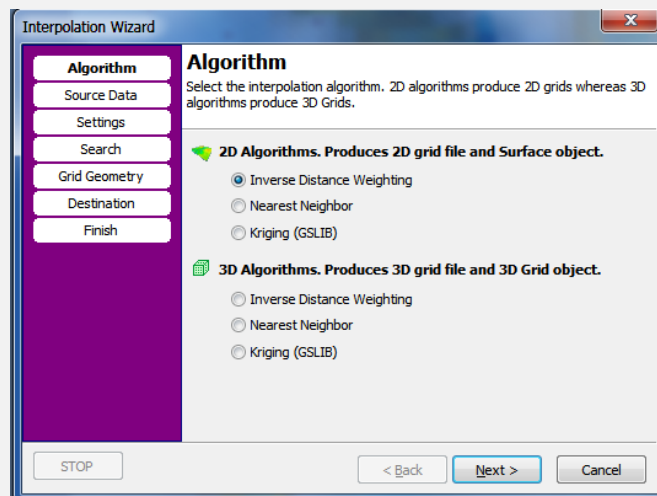
- Simple and relatively fast
- Gives nice smooth picture

Disadvantages

- Smoothing removes extremes from data



Inverse Distance Weighted in GeoScene3D



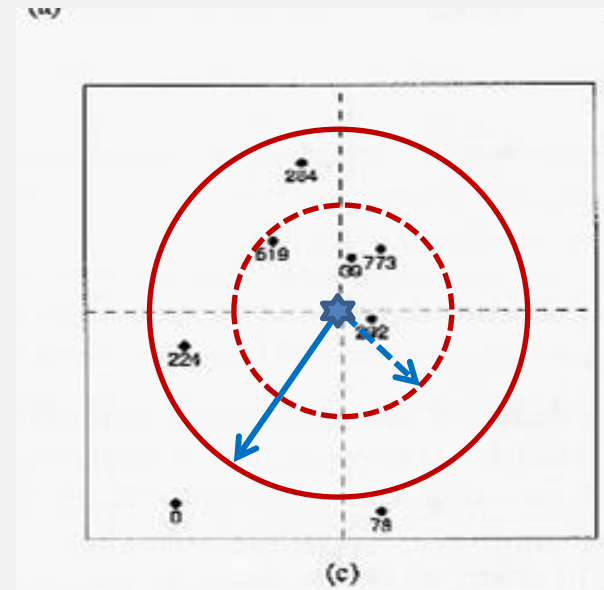
Inverse Distance Weighted in GeoScene3D

Max Count

- Defines a limit of max number of data points within a defined search radius. A kind of a “dynamic search radius”

Example:

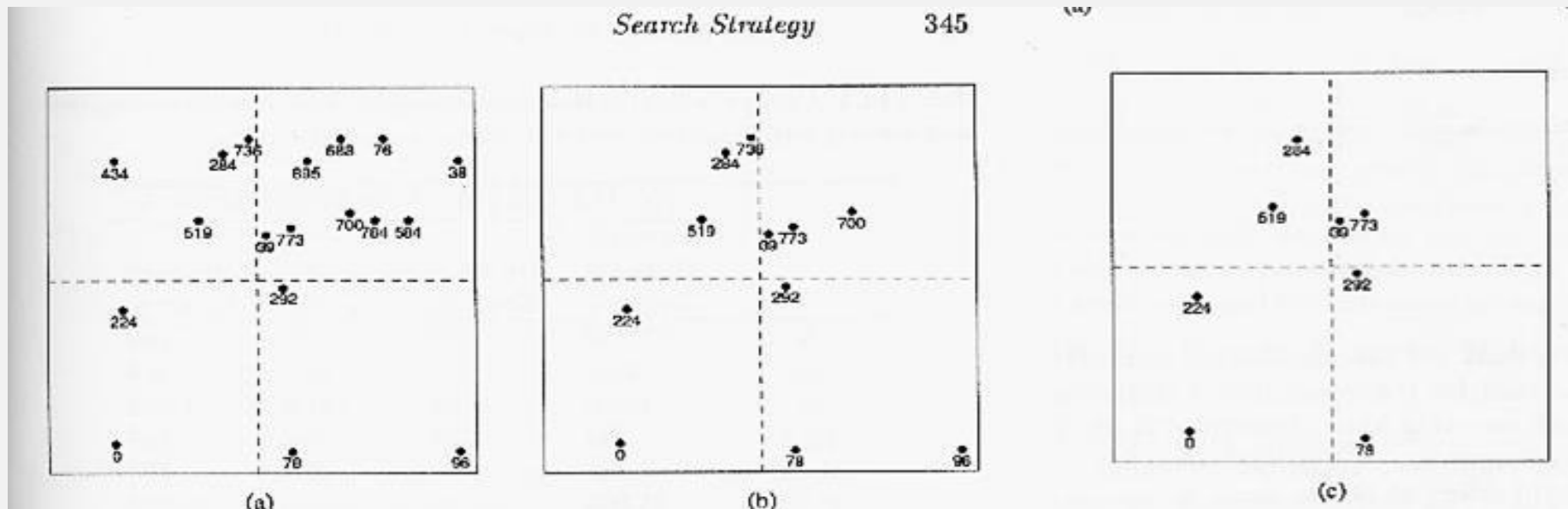
- Search Radius = 250 m
- 6 points within Search Radius
- Max Count = 4
- “Real” Search Radius will be smaller than 250 m (approx. 180 m)



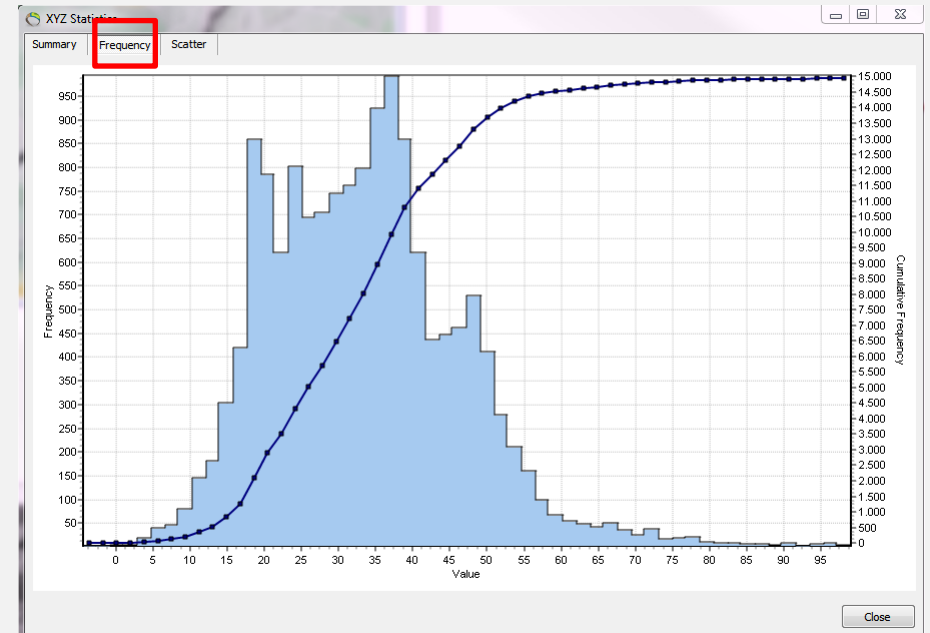
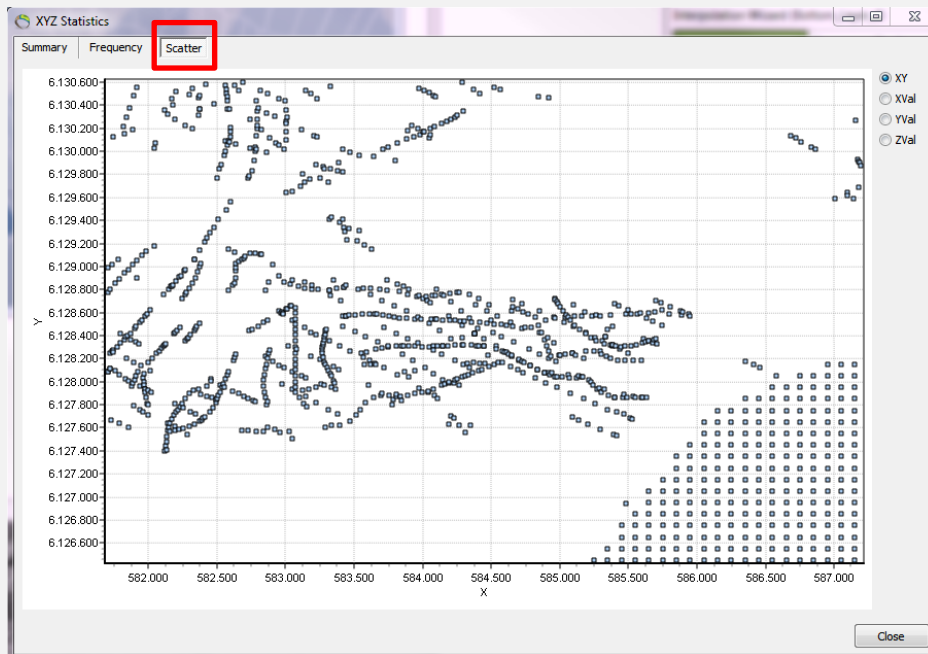
Inverse Distance Weighted in GeoScene3D

Quadrant Search

- Divide the search neighborhood into 4 quadrants and specify the maximum number of samples per quadrant
- If a quadrant has samples less than the maximum, it keeps all samples. Otherwise keeps the closest ones
- Quadrant search accomplishes some declustering



Interpolation – distribution of data points



Interpolation – Inverse Distance (example 1)

SR = 100

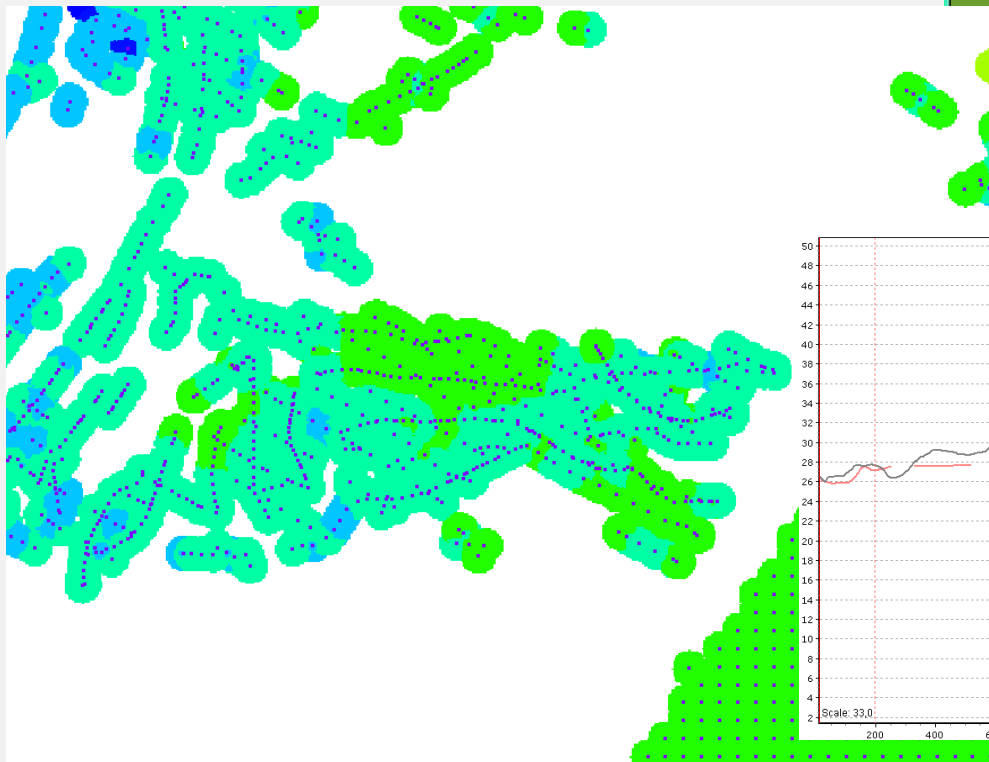
Cell Size = 100



Interpolation – Inverse Distance (example 2)

SR = 100

Cell Size = 10



Interpolation Wizard (Bottom_Layer_3)

Algorithm
Source Data
Settings
Search

Search
The search options defines what data points to consider in the interpolation. 2D algorithms search in XY only whereas 3D algorithms also search in the Z (vertical) direction.

Search Ellipsis
Angle 1: 0,00
Angle 2: 0,00
Angle 3: 0,00

Grid Geometry
Specify the size and geometry of the resulting output grid. 2D grids must have identical node spacing in XY.

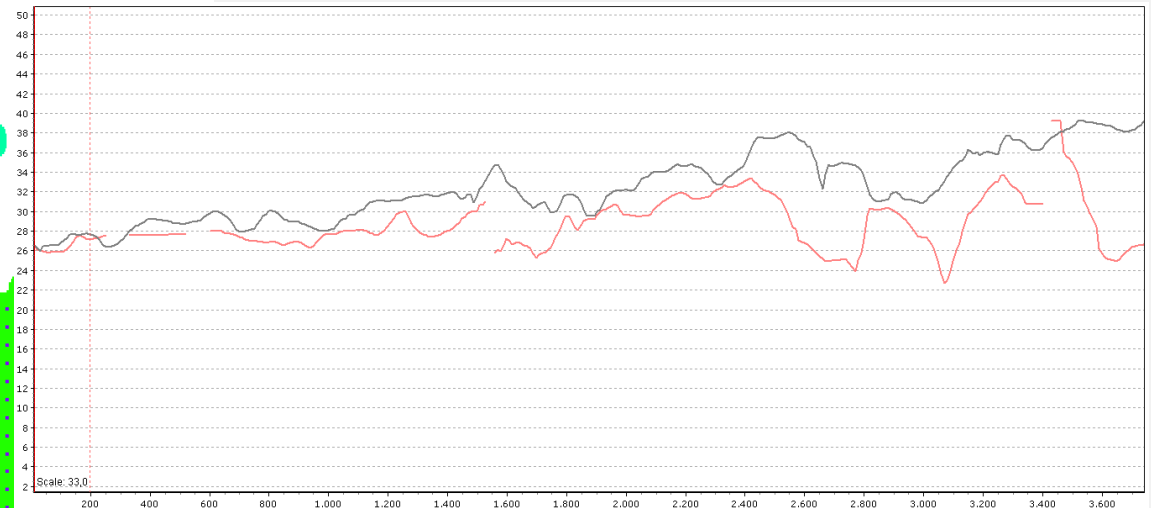
Grid Extent (corner node coordinates)
X Min: 581700,00 X Max: 587200,00
Y Min: 6126400,00 Y Max: 6130600,00
Z Min: 0,00 Z Max: 5,00

Node Spacing
X: 10,00
Y: 10,00
Z: 5,00

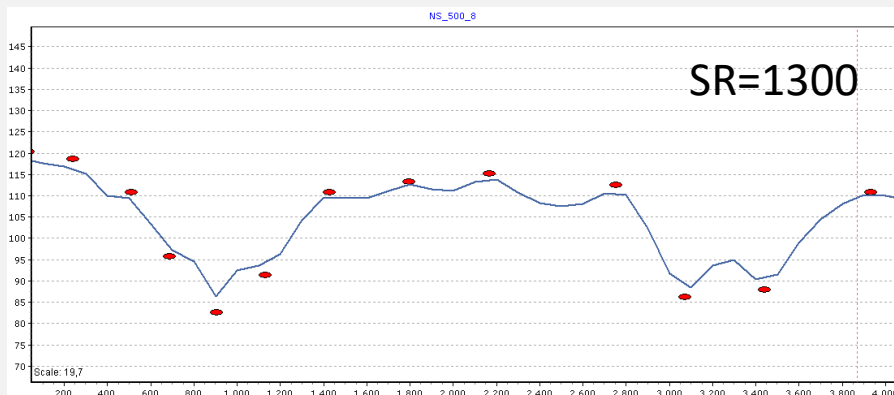
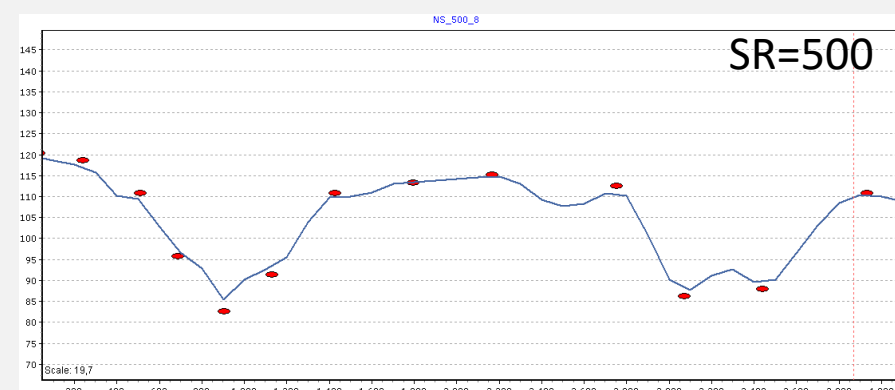
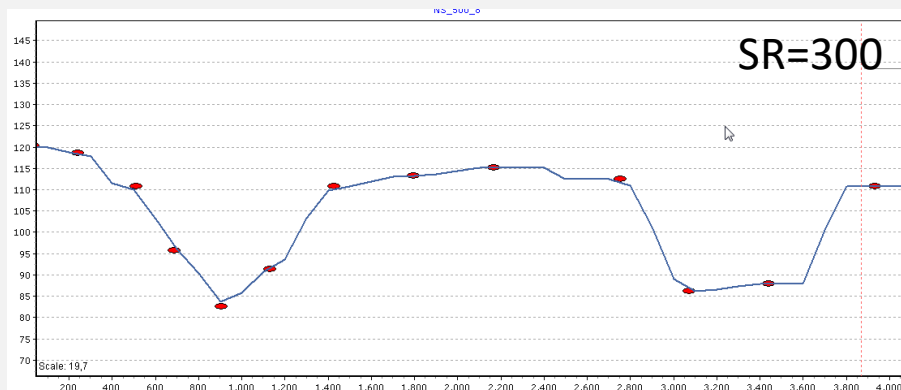
Node Count
X: 551
Y: 421
Z: 2

Grid Info
Width (X): 5500,00
Height (Y): 4200,00
Depth (Z): 5,00
Nodes: 231971

< Back Next > Cancel



Interpolation – Inverse Distance: *Search Radius*



$$w_i(x) = \frac{1}{(\sqrt{(x - x_i)^2 + (y - y_i)^2 + s^2})^p}$$

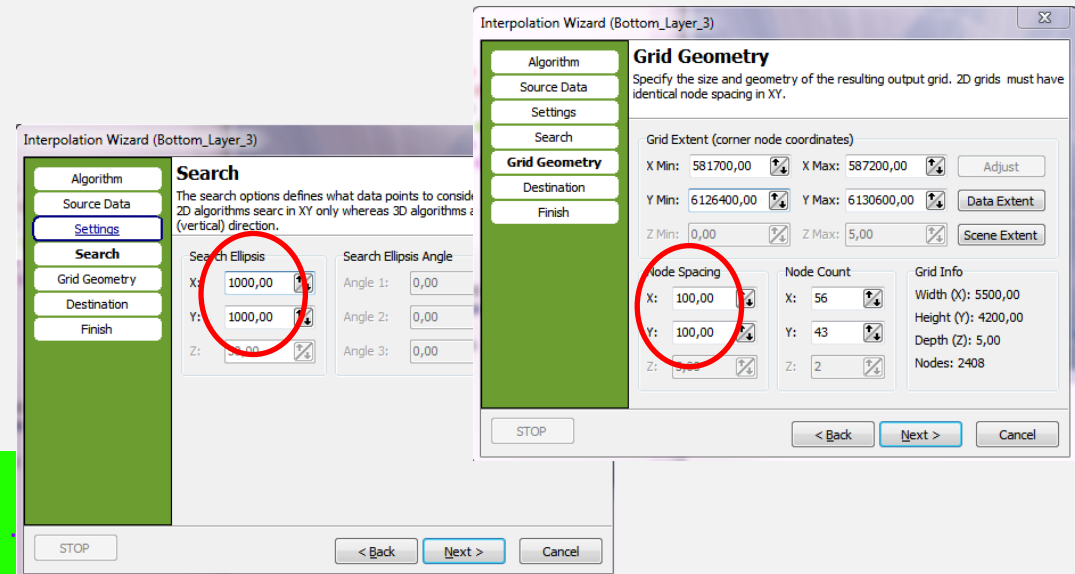
- As search radius increases more data are included in the interpolation
- Weighting has an exponential growth as we approach the location of data points
- Smoothing counteracts this...

Interpolation – Inverse Distance (example 3)

SR = 1000

Cell Size = 100

Point Limit = None



Interpolation Wizard (Bottom_Layer_3)

Search

The search options defines what data points to consider. 2D algorithms search in XY only whereas 3D algorithms search in XYZ (vertical) direction.

Search Ellipsis

X: 1000,00
Y: 1000,00
Z: 0,00

Search Ellipsis Angle

Angle 1: 0,00
Angle 2: 0,00
Angle 3: 0,00

Grid Geometry

Specify the size and geometry of the resulting output grid. 2D grids must have identical node spacing in XY.

Grid Extent (corner node coordinates)

X Min: 581700,00 X Max: 587200,00
Y Min: 6126400,00 Y Max: 6130600,00
Z Min: 0,00 Z Max: 5,00

Node Spacing

X: 100,00
Y: 100,00
Z: 0,00

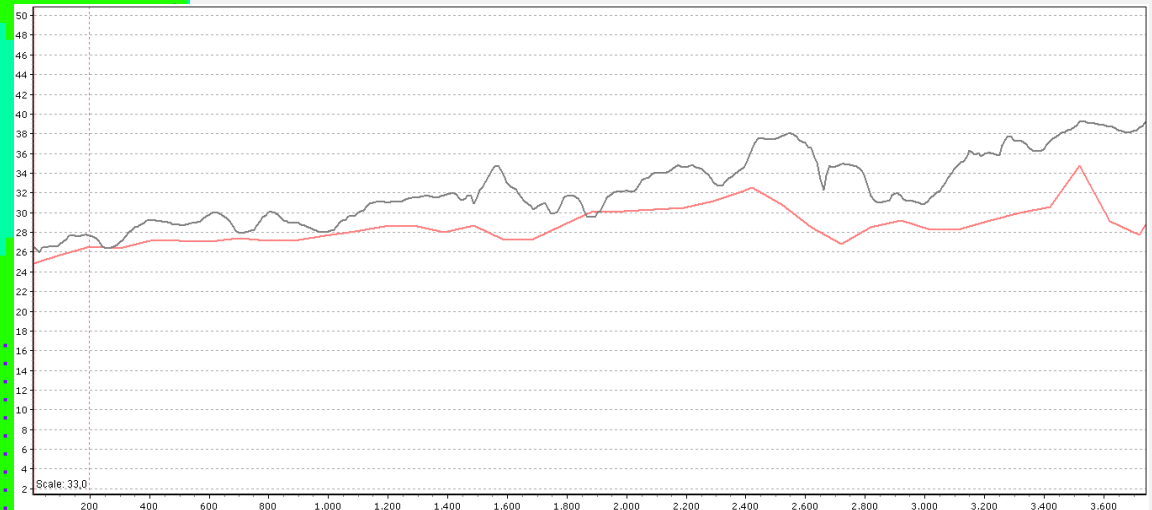
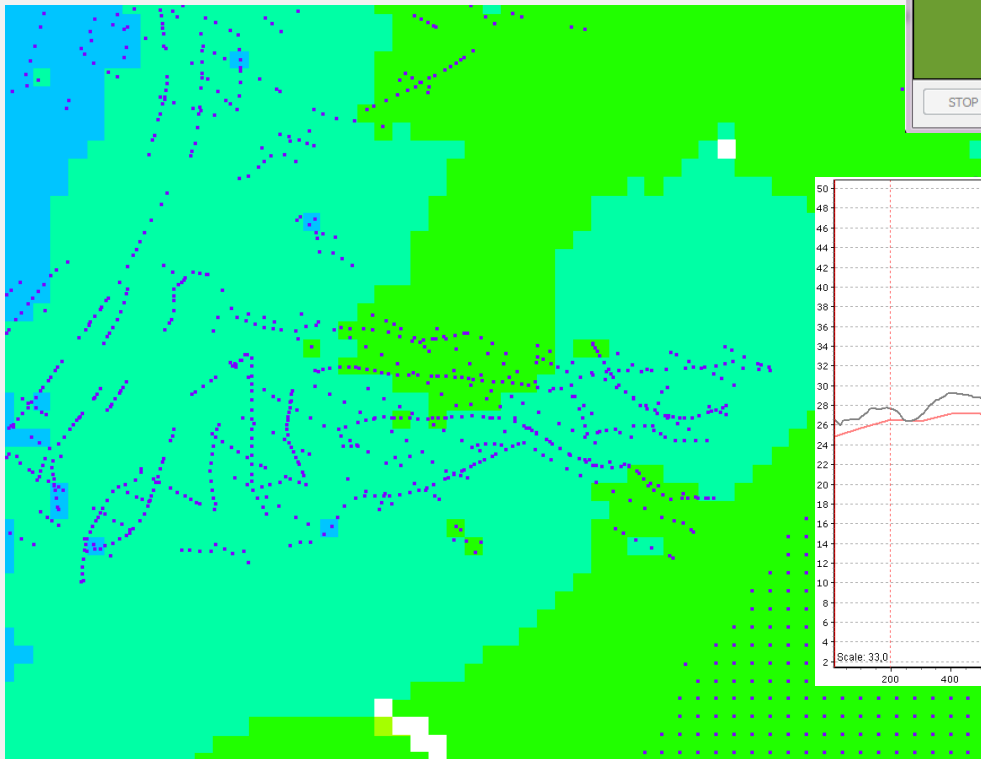
Node Count

X: 56
Y: 43
Z: 2

Grid Info

Width (X): 5500,00
Height (Y): 4200,00
Depth (Z): 5,00
Nodes: 2408

STOP < Back Next > Cancel

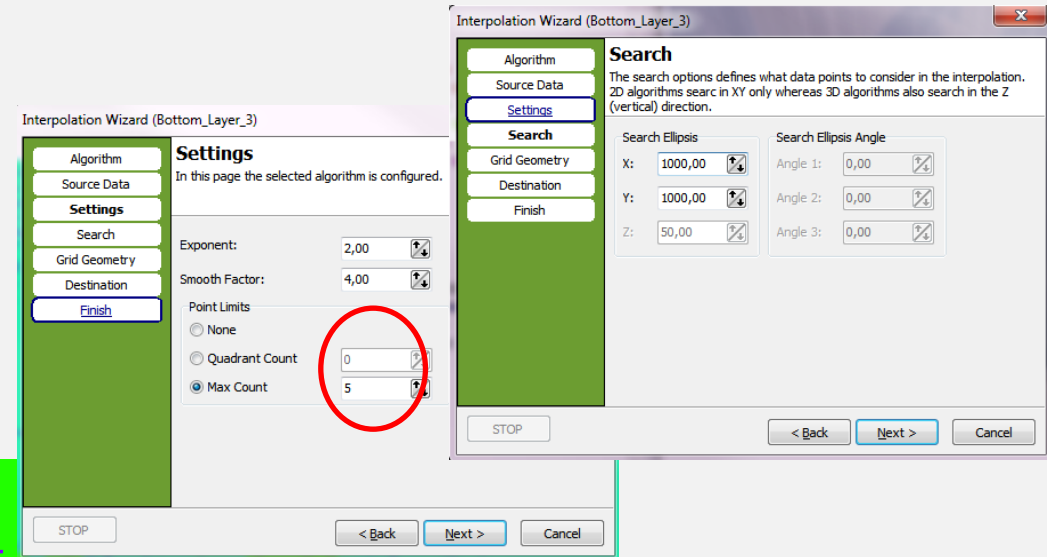


Interpolation – Inverse Distance (example 4)

SR = 1000

Cell Size = 100

Point Limit = 5



Interpolation Wizard (Bottom_Layer_3)

Settings
In this page the selected algorithm is configured.

Exponent: 2,00

Smooth Factor: 4,00

Point Limits

- ☐ None
- ☐ Quadrant Count
- ☒ Max Count

0

5

Interpolation Wizard (Bottom_Layer_3)

Search
The search options defines what data points to consider in the interpolation. 2D algorithms search in XY only whereas 3D algorithms also search in the Z (vertical) direction.

Search Ellipsoid

X: 1000,00

Y: 1000,00

Z: 50,00

Search Ellipsoid Angle

Angle 1: 0,00

Angle 2: 0,00

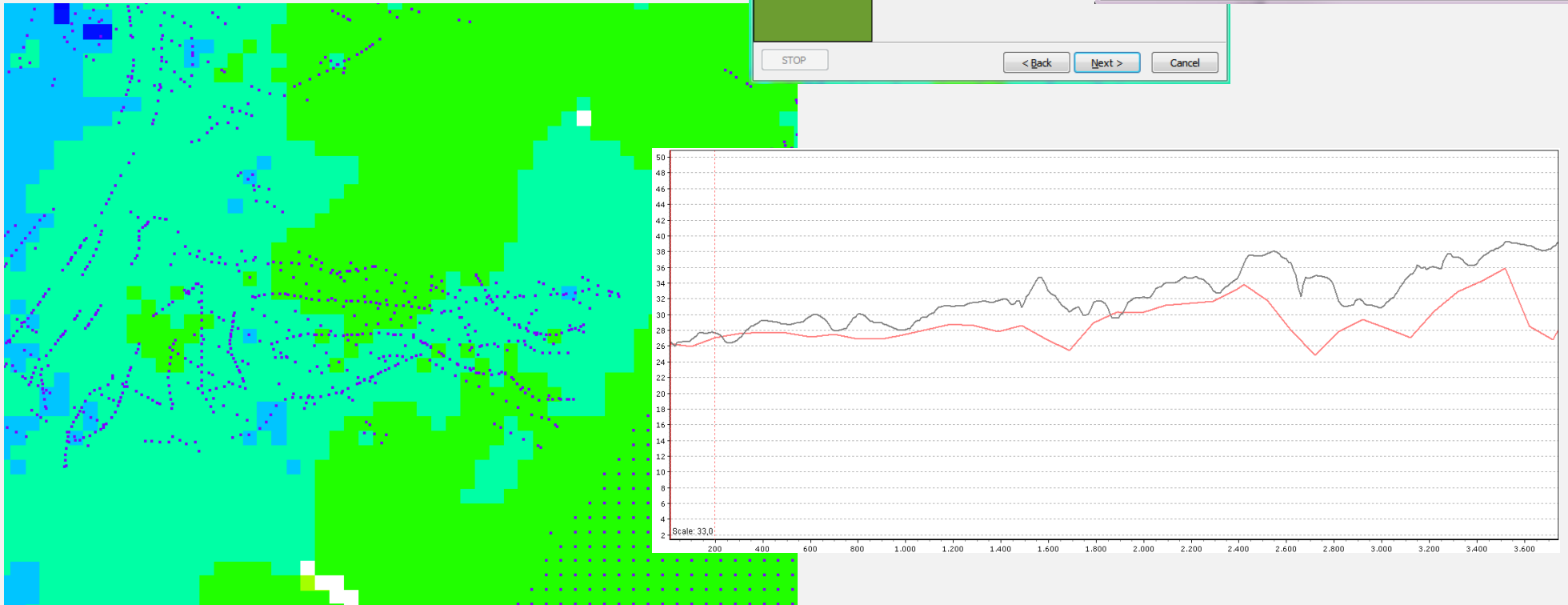
Angle 3: 0,00

STOP

< Back

Next >

Cancel



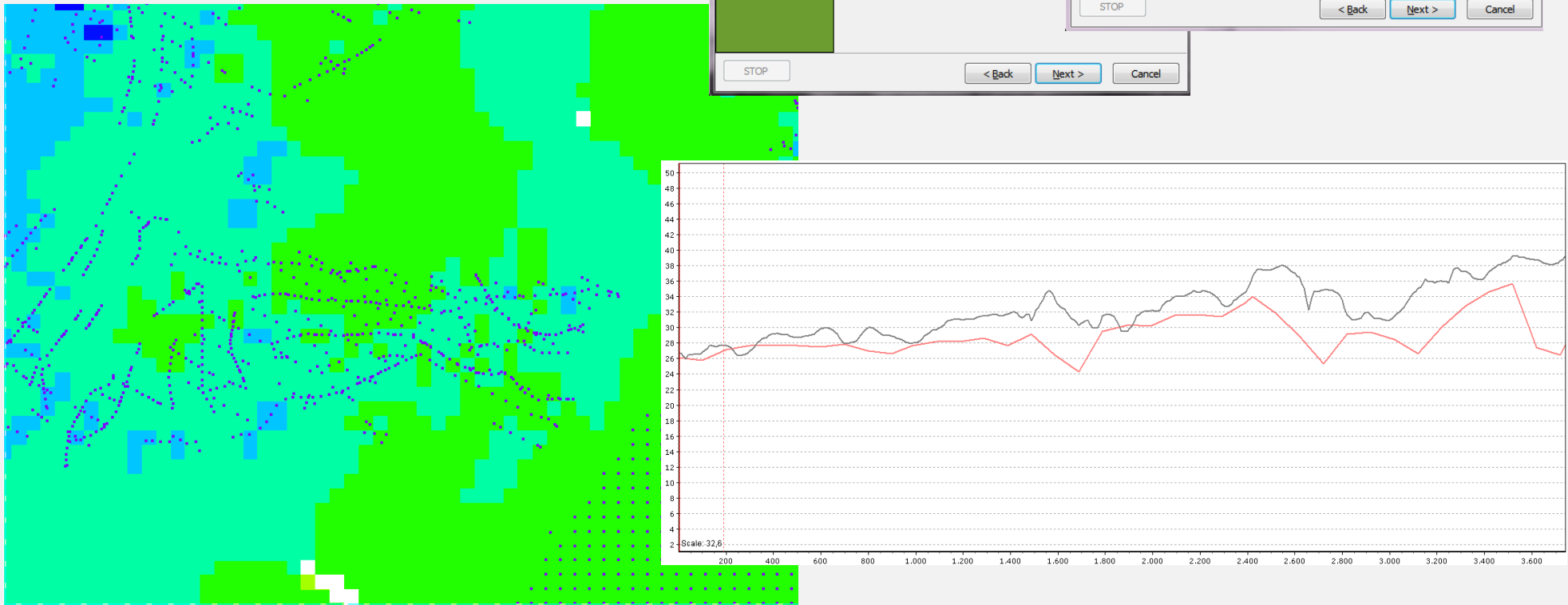
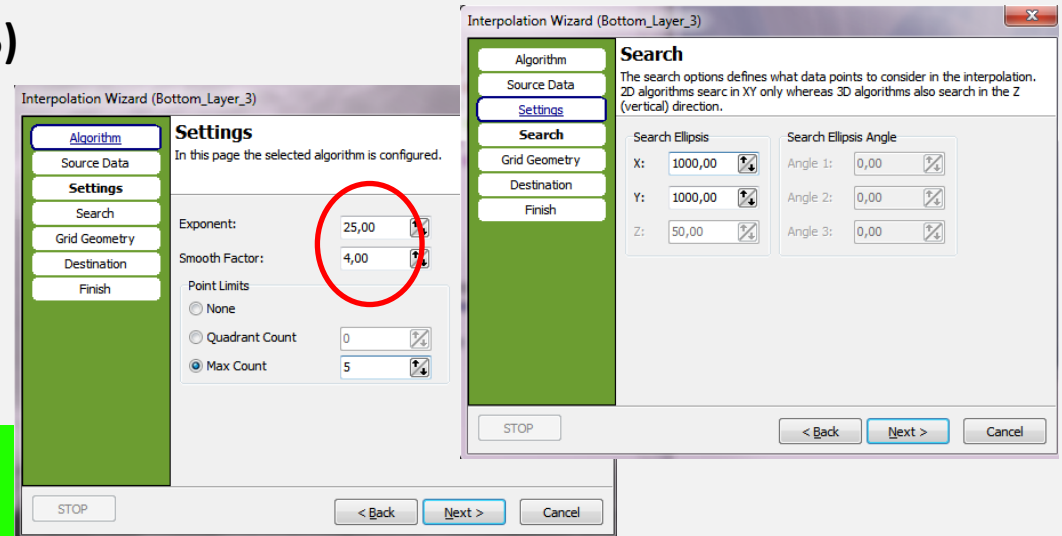
Interpolation – Inverse Distance (example 5)

SR = 1000

Cell Size = 100

Point Limit = 5

Exponent = 25



Interpolation – Inverse Distance: *Smooth Factor*

IDW General definition:

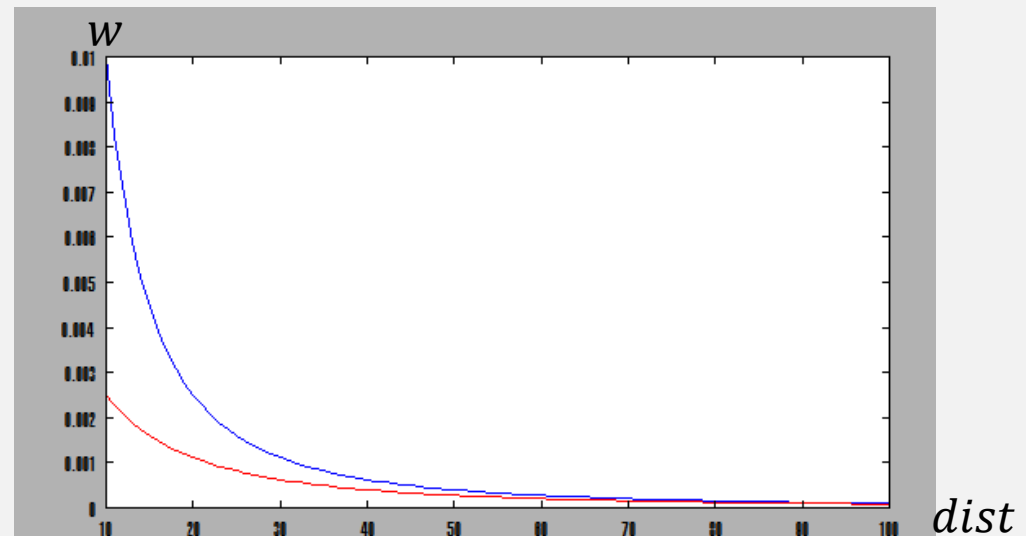
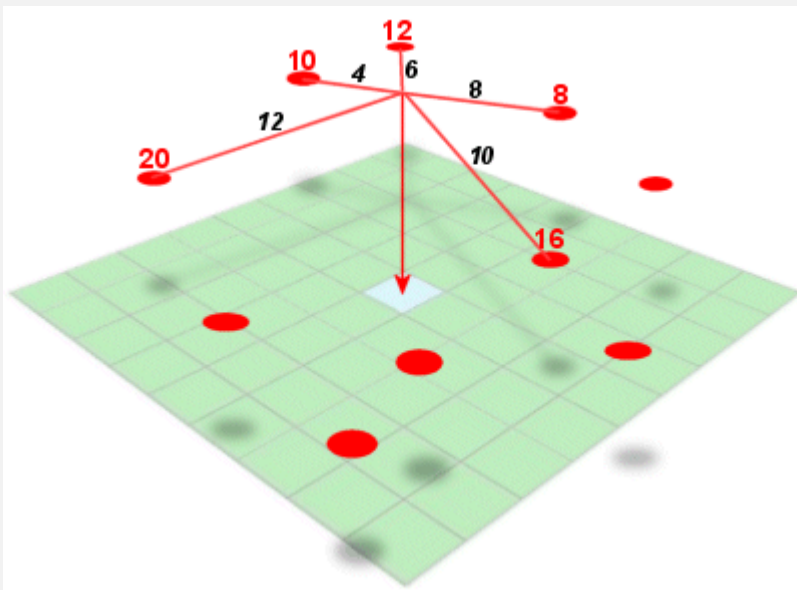
Interpolate a value, u , at a given point $\mathbf{x}$ based on samples for $u_i = u(x_i)$ for $i = 0, 1, \dots, N$

$$u(\mathbf{x}) = \sum_{i=0}^N \frac{w_i(\mathbf{x})u_i}{\sum_{j=0}^N w_j(\mathbf{x})}, \quad \text{where weighting function is: } w_i(\mathbf{x}) = \frac{1}{d(\mathbf{x}, \mathbf{x}_i)^p}$$

In GeoScene3D, a smoothing factor, s , is introduced in the weighting function as:

$$w_i(x) = \frac{1}{(\sqrt{(x - x_i)^2 + (y - y_i)^2 + s^2})^p}$$

S: Makes weight smaller close to x .



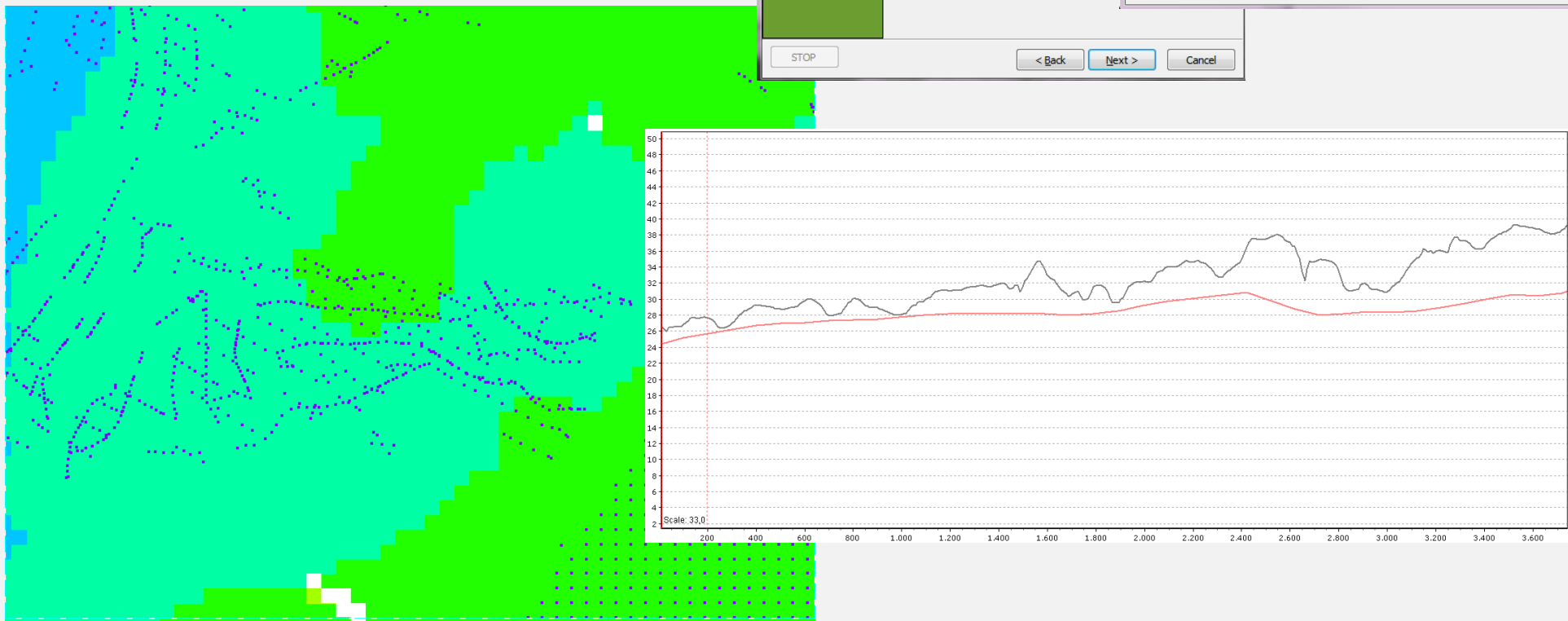
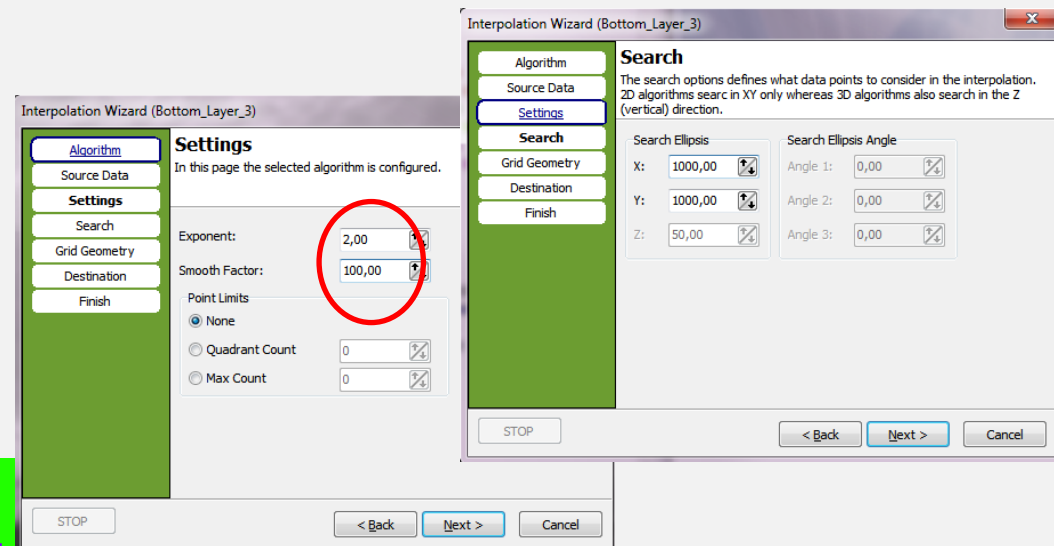
Interpolation – Inverse Distance (example 6)

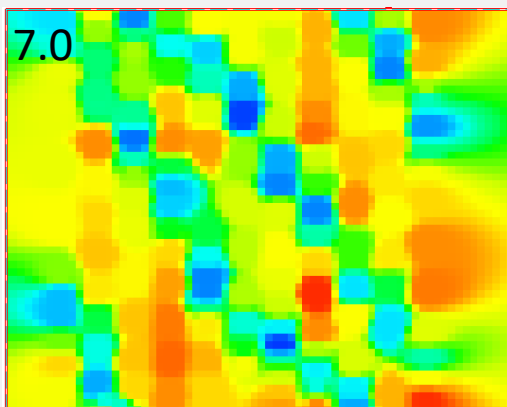
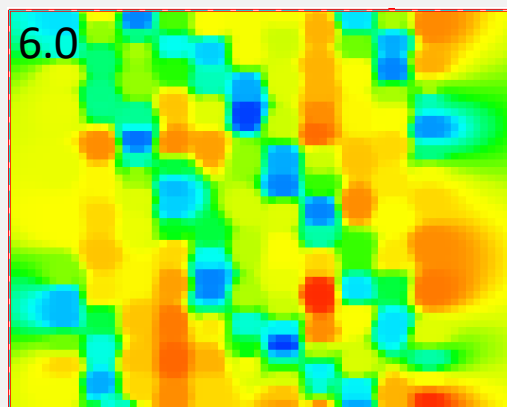
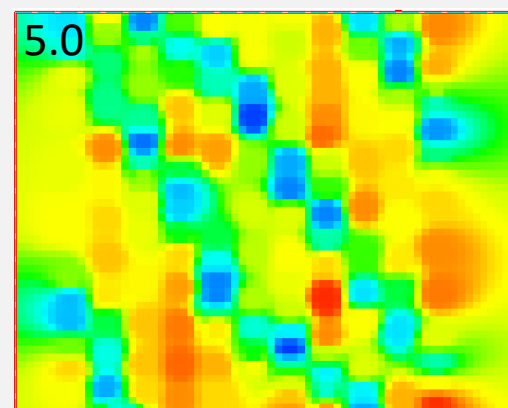
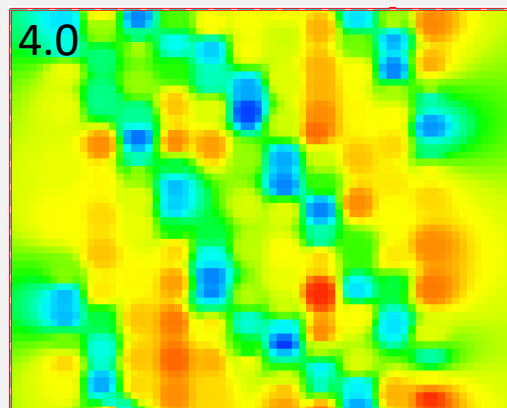
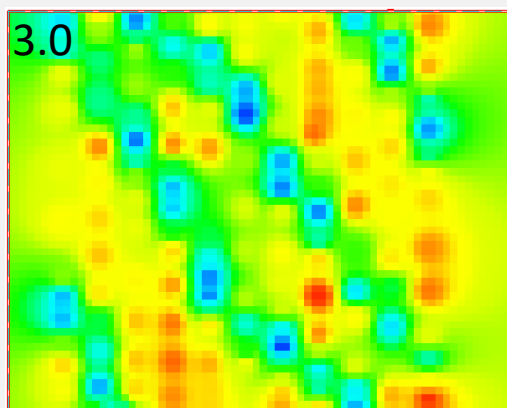
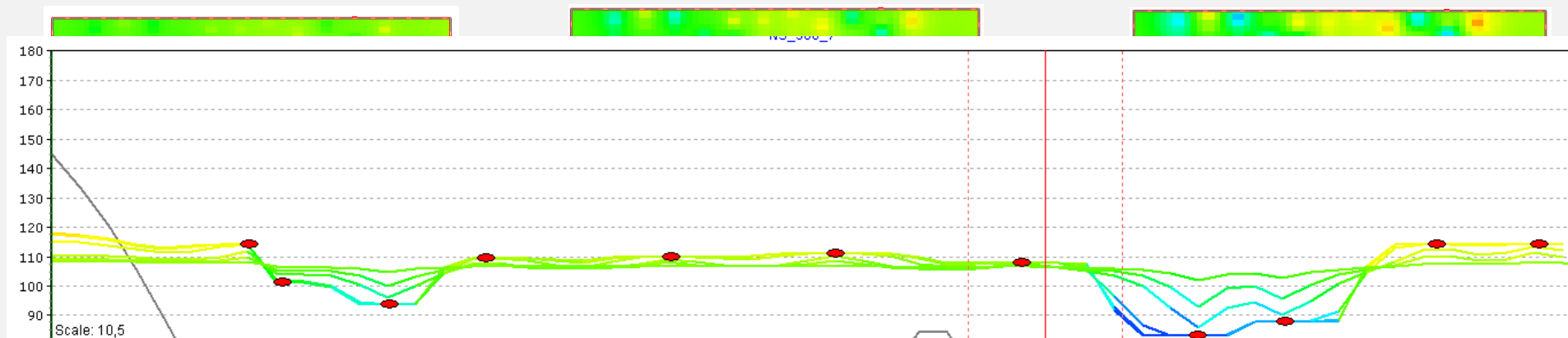
SR = 1000

Cell Size = 100

Point Limit = None

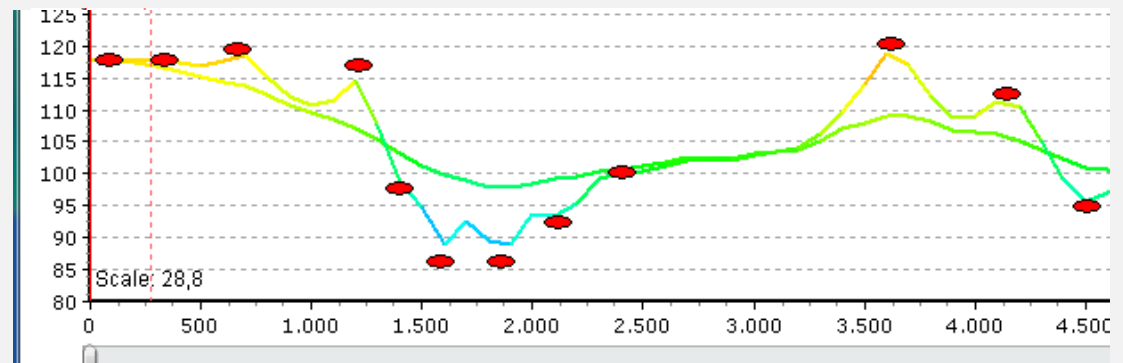
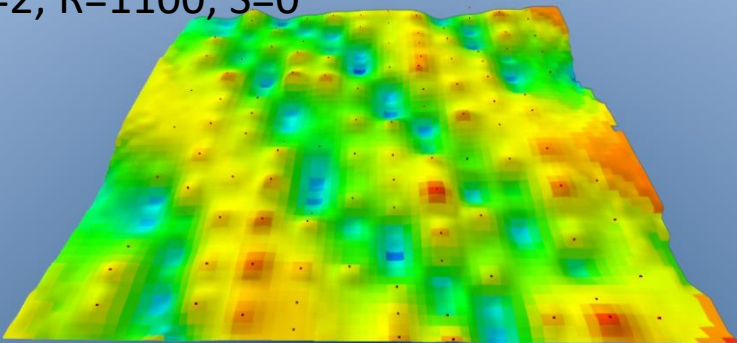
Smooth Factor = 100



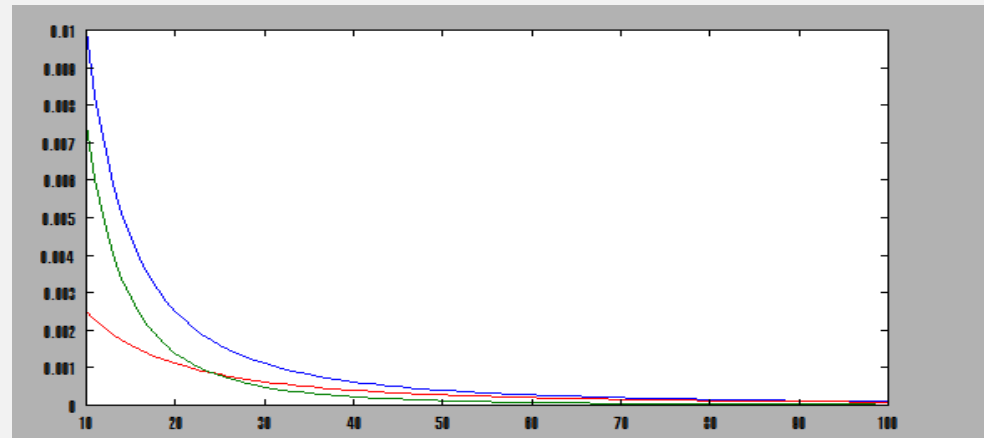
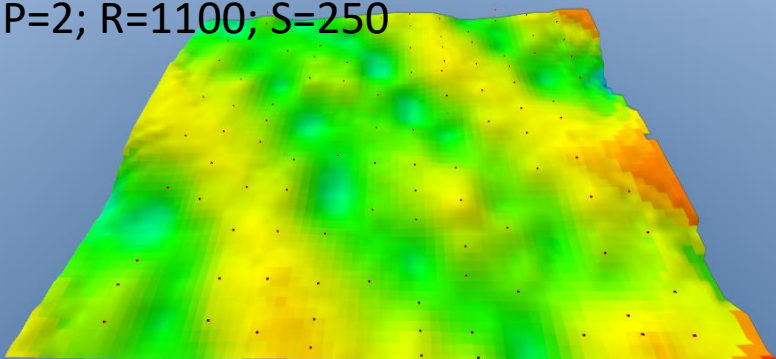


Different Exponents

P=2; R=1100; S=0

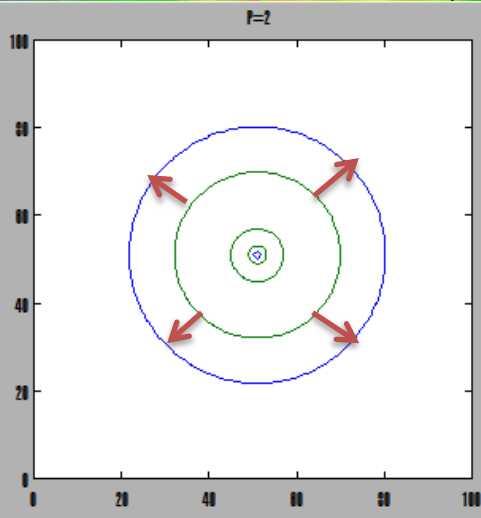


P=2; R=1100; S=250



- Smoothing factor counteracts this – see formula:

$$w_i(x) = \frac{1}{(\sqrt{(x - x_i)^2 + (y - y_i)^2} + s^2)^p}$$



Inverse Distance Weighting

- Works best for dense, evenly-spaced sample point sets.
- Does not consider any trends in the data
 - *if actual surface values change more in the north-south direction than they do in the east-west direction the interpolated surface will average out this bias rather than preserve it (Quadrant Count can to some extent compensate for this).*
- Cannot make estimates above the maximum or below the minimum sample values.
For an elevation surface, this has the effect of flattening peaks and valleys (unless their high and low points are part of the sample).
- Interpolated value will correspond to data value in data point – if grid and point are overlapping.
- Smooth Factor counteracts spiking...
- Check your search radius if experiencing interpolation artifacts...
- Check your data distribution in the statistics – high differences might call for a higher Exponent...

Agenda

- Interpolation defined
- General workflow in GeoScene3D
- Interpolation methods: Nearest-Neighbour (NN)
- Interpolation methods: Inverse Distance Weighting
- **Interpolation methods: Kriging**
- Summing up

Kriging

Description

At each grid point a value is assigned based on a weighted using a statistical approach. Uses GSLIB.

Parameters

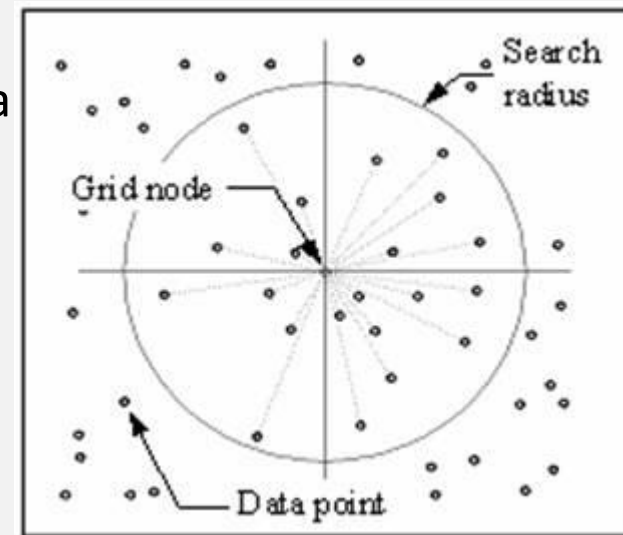
- MANY !! 😊

Advantages

- Based on statistics
- Supplies a measure of uncertainty – variance grid.
- A statistical approach

Disadvantages

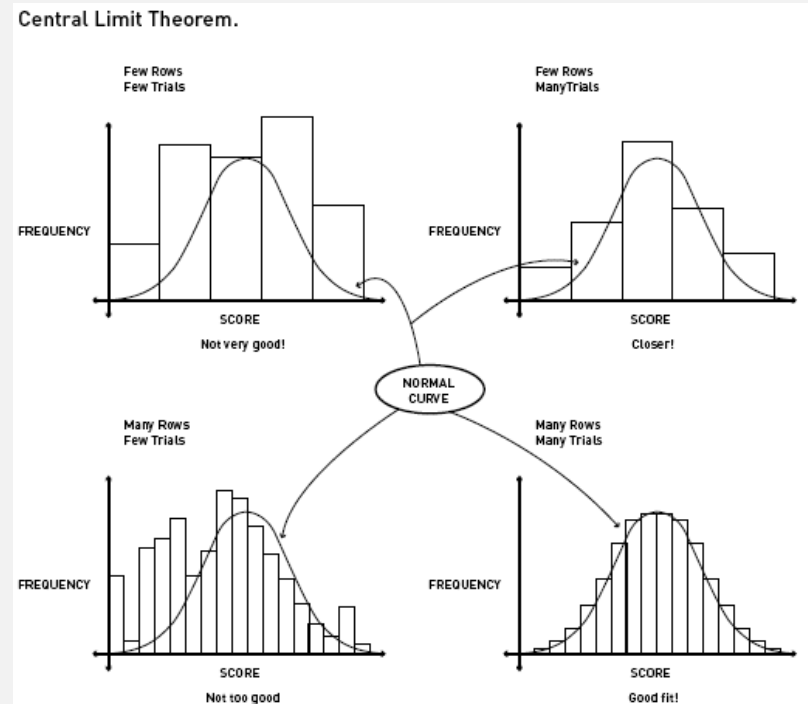
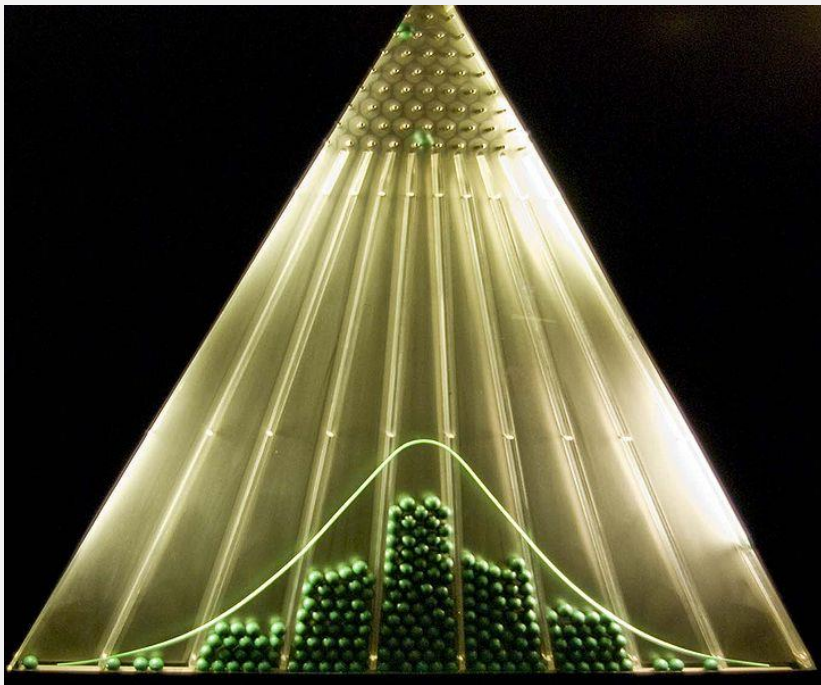
- Time consuming – slow performance
- Complicated – requires knowledge !!



Data must have Normal – Gaussian - Distribution

Gauss Distribution and Central Limit Theorem

Many physical phenomena in nature arise from a number of additive variations. The distributions of the individual variations are often unknown, however, the histogram of the summed variable is often observed to be approximately Normal.



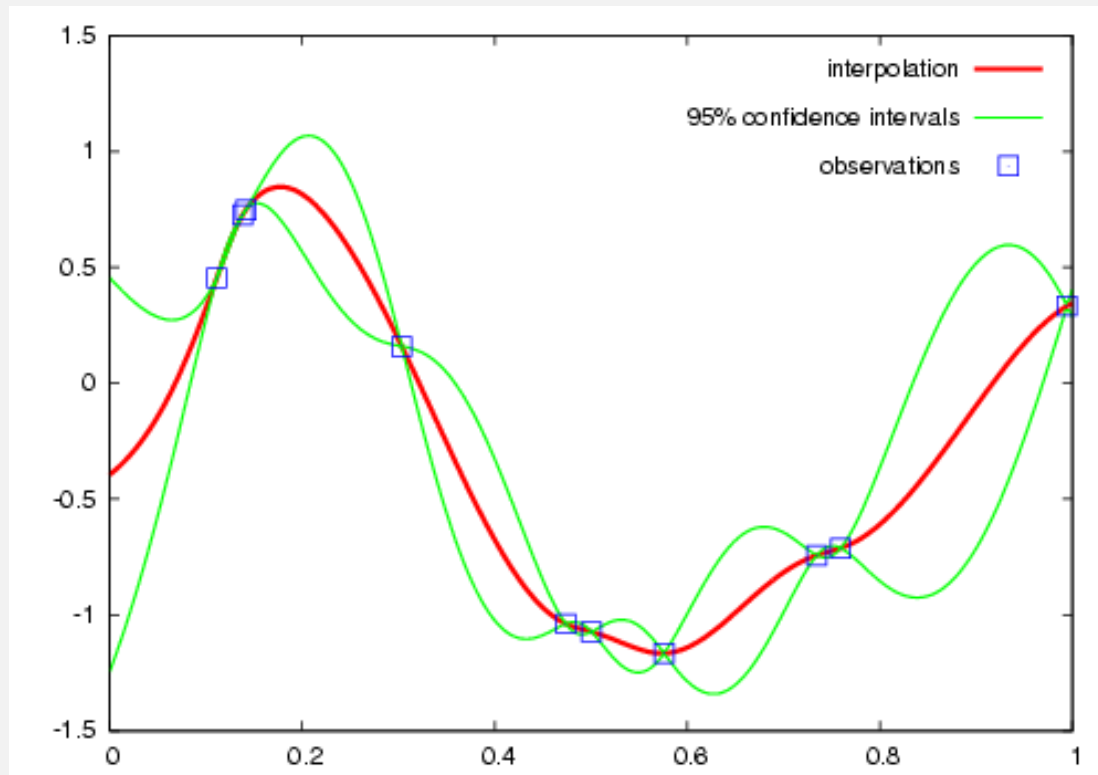
SAMPLING FREQUENCY !!

Example

Bean Machine – beans are dropped and randomly falls into binns. As by magic the Gaussian distribution appears ☺

Kriging

Interpolation based on an assumption that the data describes the sampling of a normally (Gaussian) distributed phenomena.



We get:

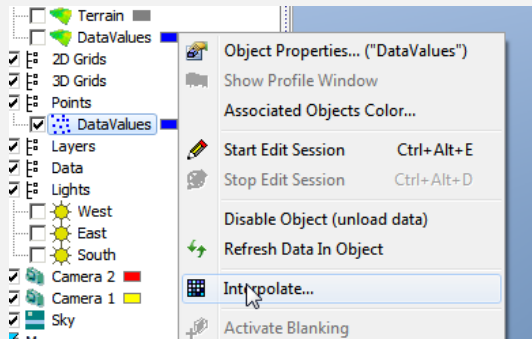
- A value estimate
- An uncertainty estimate

All based on the assumption of a GAUSSIAN distribution of our data !!!

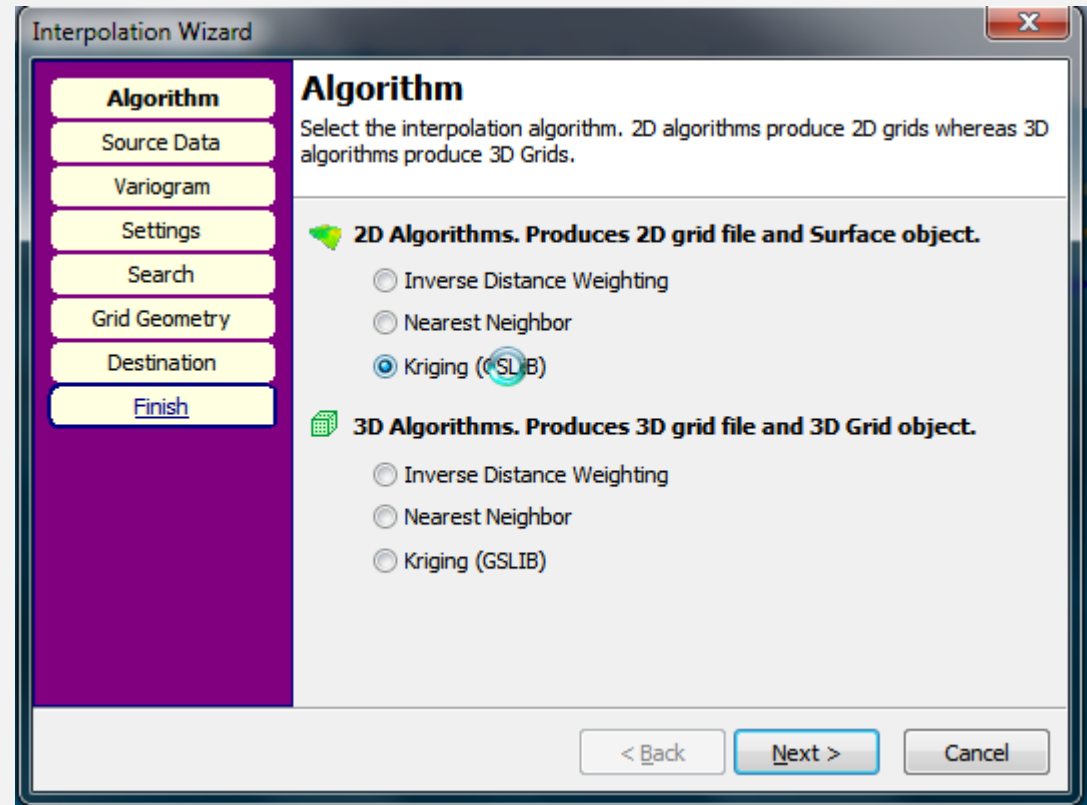
Kriging example in 1 Dimension

Example of one-dimensional data interpolation by kriging, with confidence intervals. Squares indicate the location of the data. The kriging interpolation is in red. The confidence intervals are in green

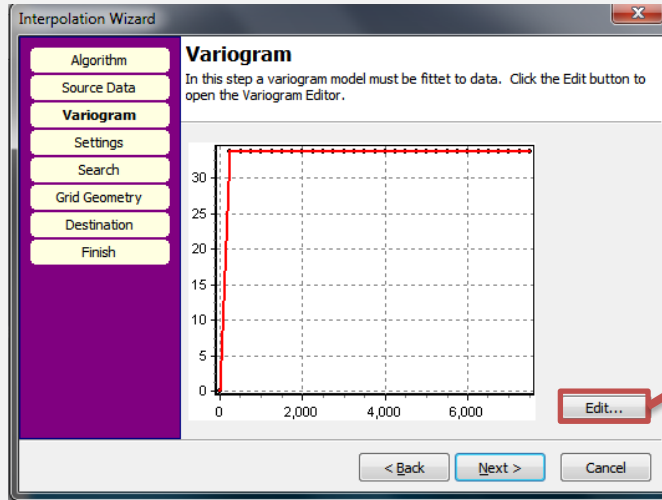
Kriging User Inter face and methodology in GeoScene3D



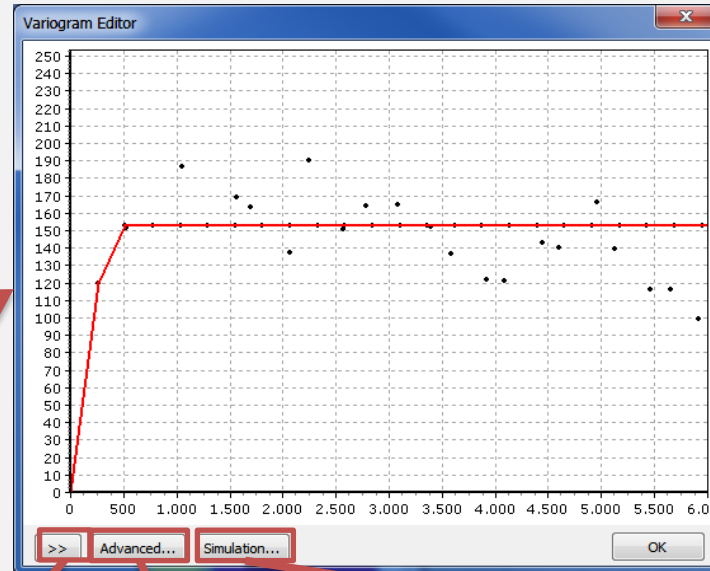
Activate



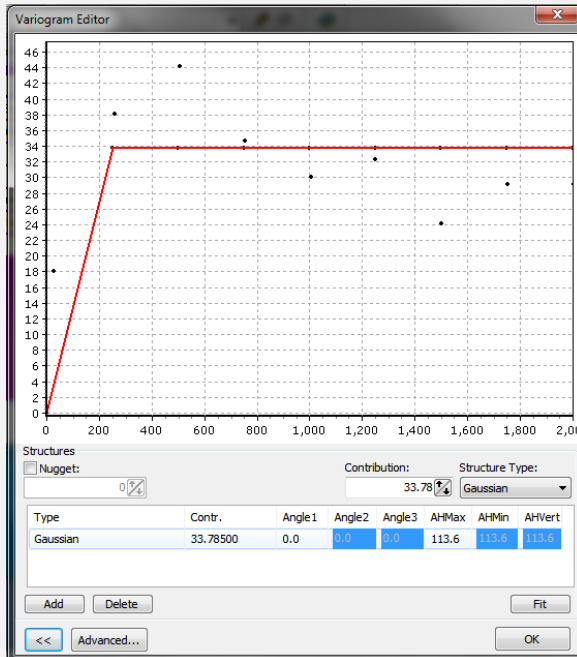
Variogram



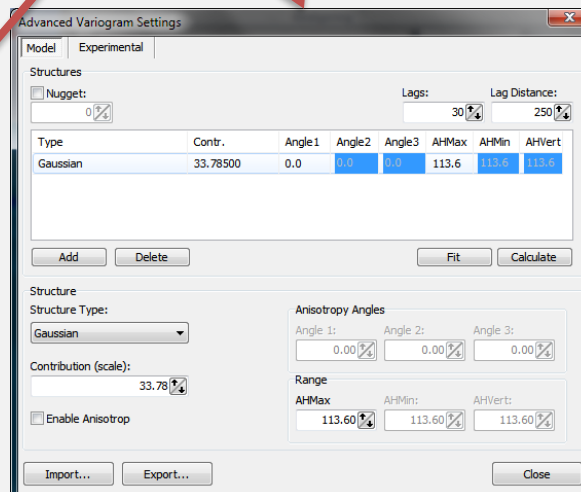
Variogram Editor - collapsed



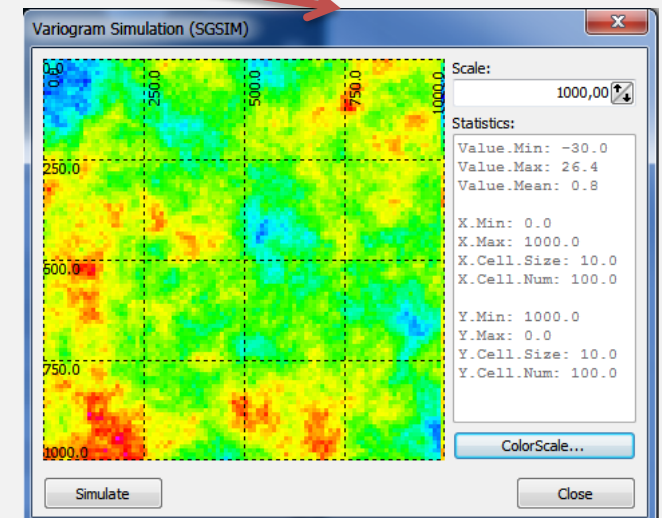
Expanded



Advanced



Simulation

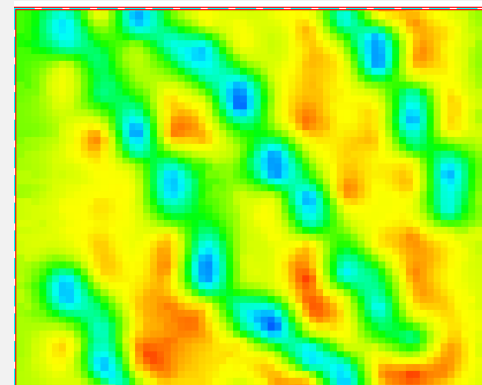
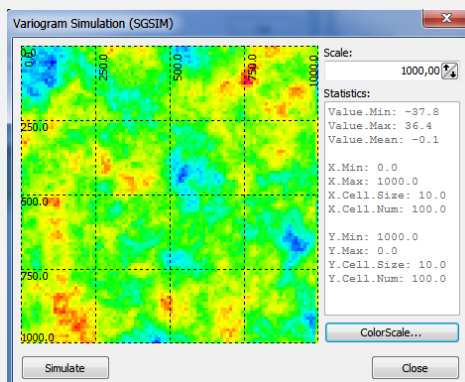
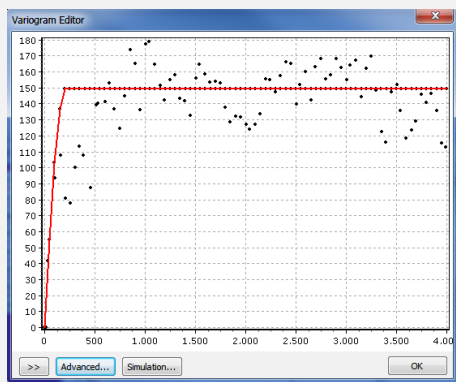


Variogram

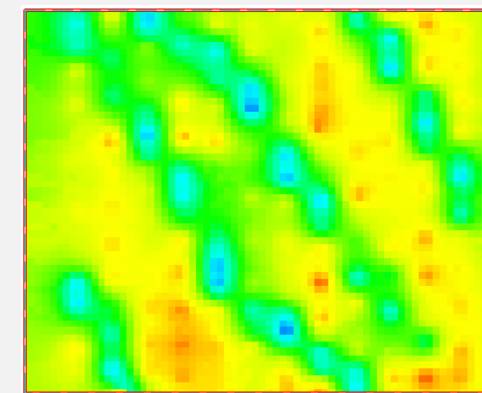
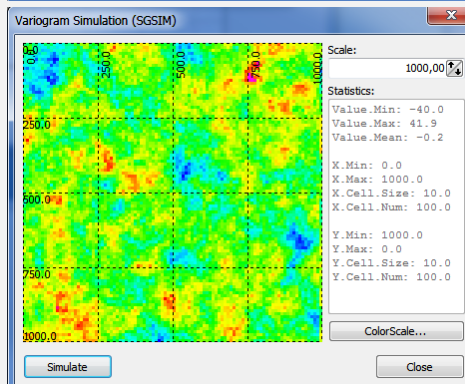
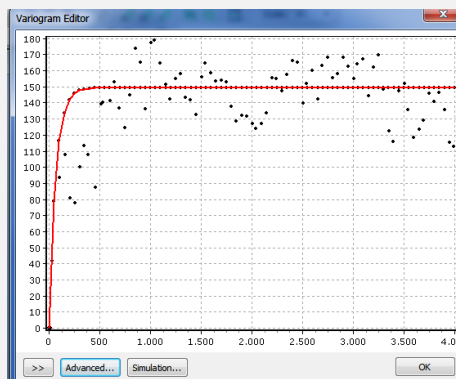
Simulation

Kriging Result

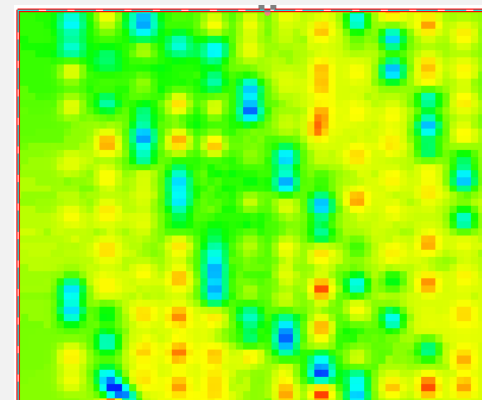
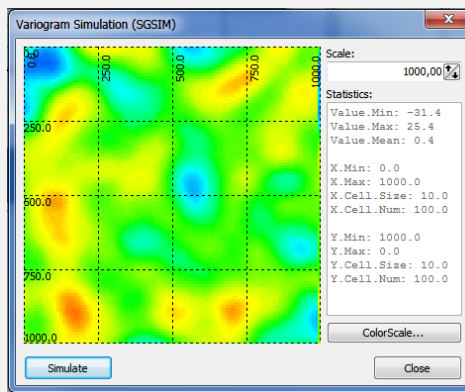
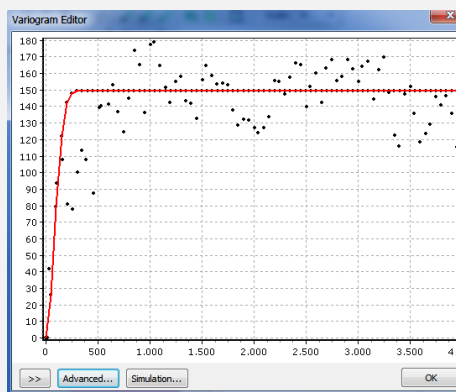
Spherical



Exponential



***Gaussian**



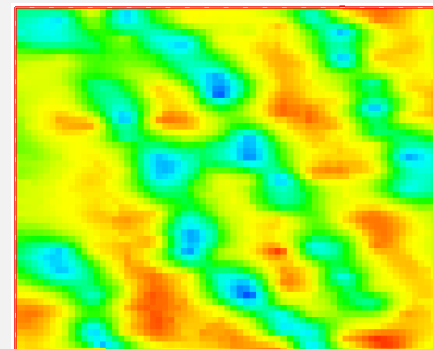
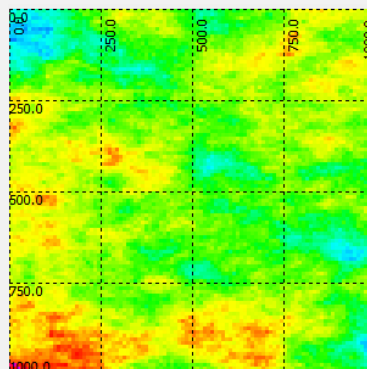
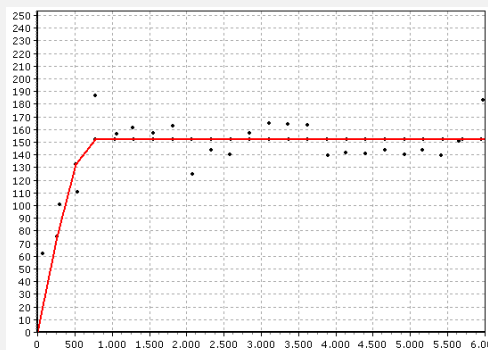
**NOTE: a nugget of 0.01 added!*

Variogram

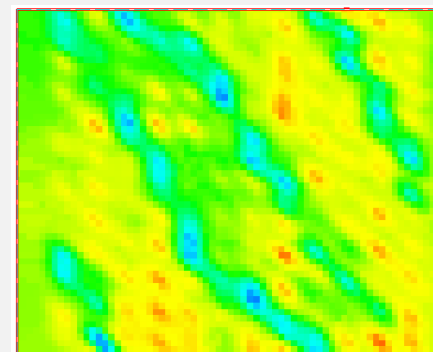
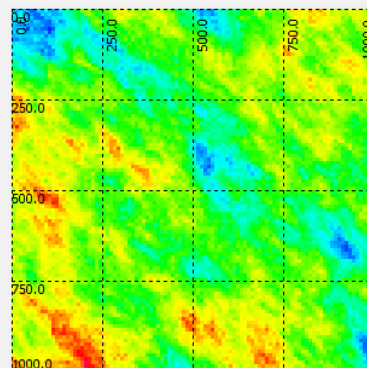
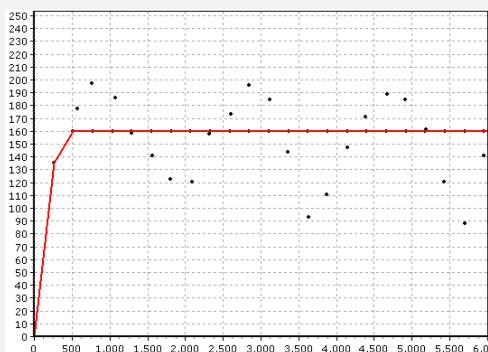
Simulation

Kriging Result

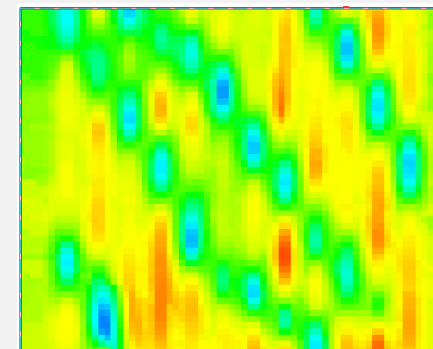
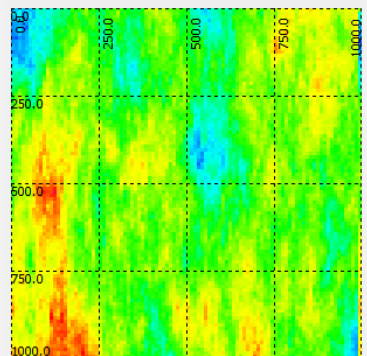
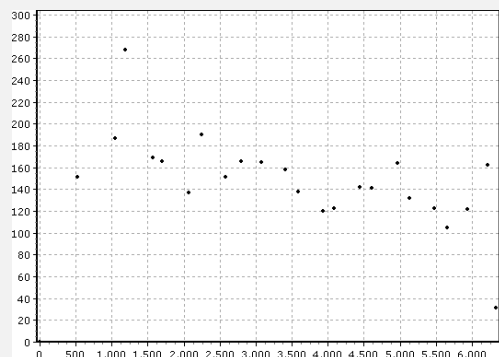
AZ=0



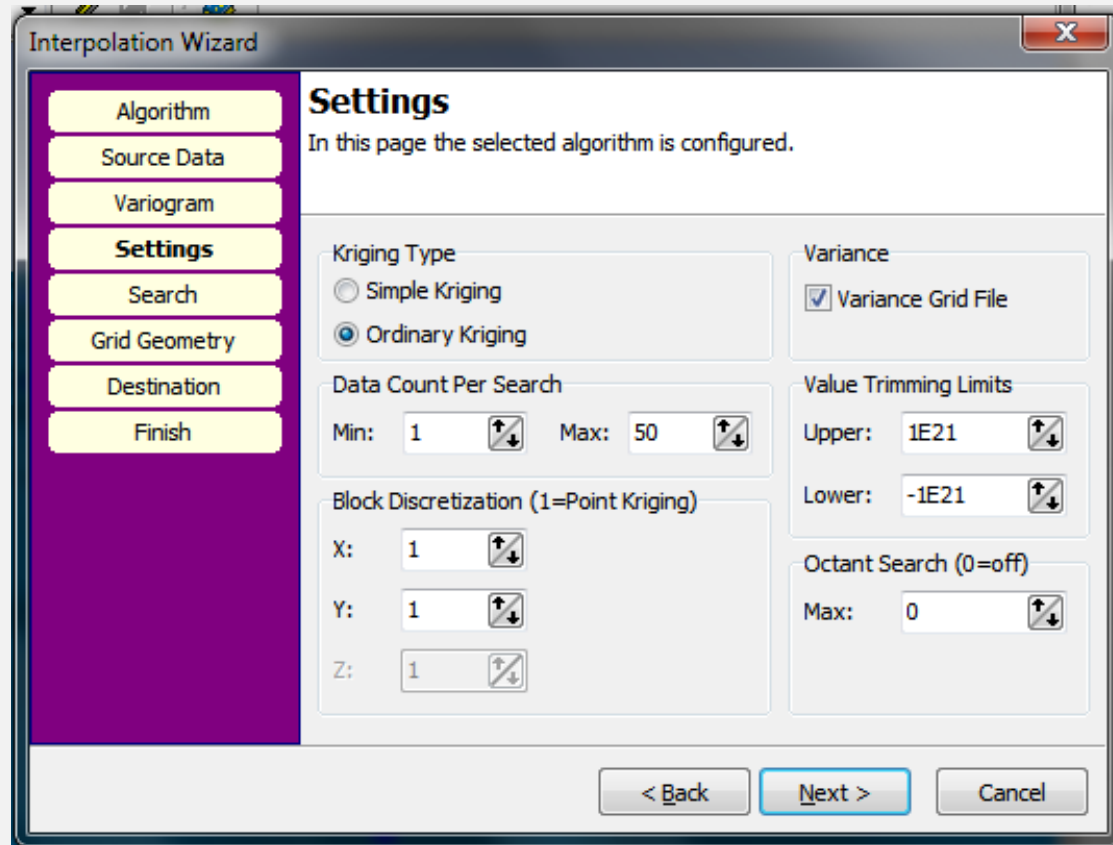
AZ=45



AZ=90



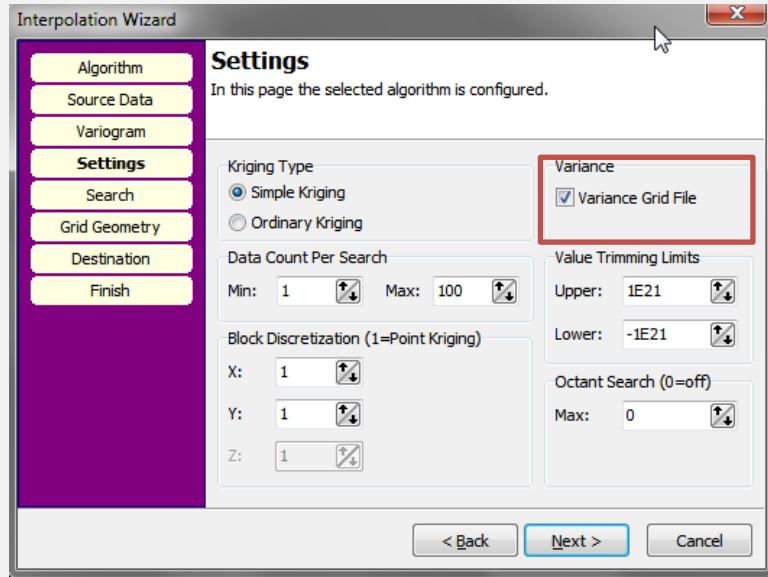
Settings



The image shows a software dialog box titled "Interpolation Wizard" with a close button (X) in the top right corner. On the left is a vertical sidebar with a purple background and yellow buttons for the wizard steps: "Algorithm", "Source Data", "Variogram", "Settings" (which is highlighted), "Search", "Grid Geometry", "Destination", and "Finish". The main area is titled "Settings" and contains the text "In this page the selected algorithm is configured." Below this, there are several configuration sections:

- Kriging Type:** Two radio buttons, "Simple Kriging" and "Ordinary Kriging" (which is selected).
- Variance:** A checkbox labeled "Variance Grid File" which is checked.
- Data Count Per Search:** Two spin boxes, "Min:" (set to 1) and "Max:" (set to 50).
- Value Trimming Limits:** Two spin boxes, "Upper:" (set to 1E21) and "Lower:" (set to -1E21).
- Block Discretization (1=Point Kriging):** Three spin boxes for "X:", "Y:", and "Z:", all set to 1.
- Octant Search (0=off):** A spin box for "Max:" set to 0.

At the bottom of the dialog are three buttons: "< Back" (disabled), "Next >" (active/highlighted), and "Cancel" (disabled).

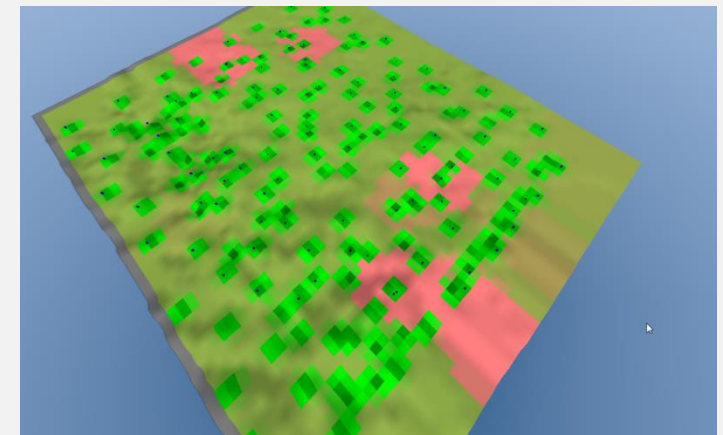
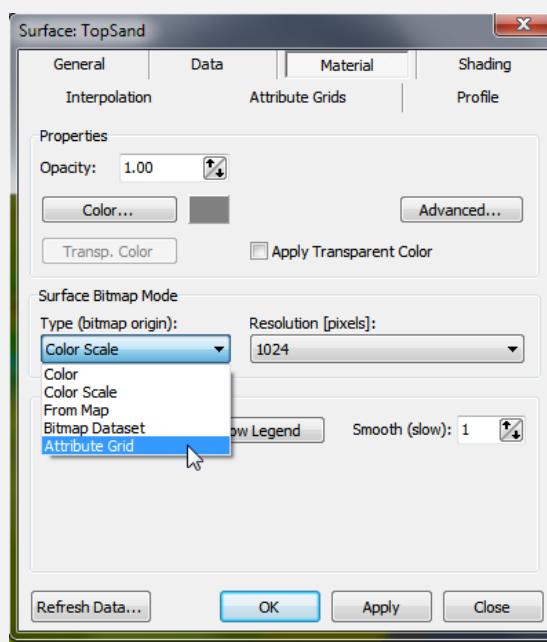
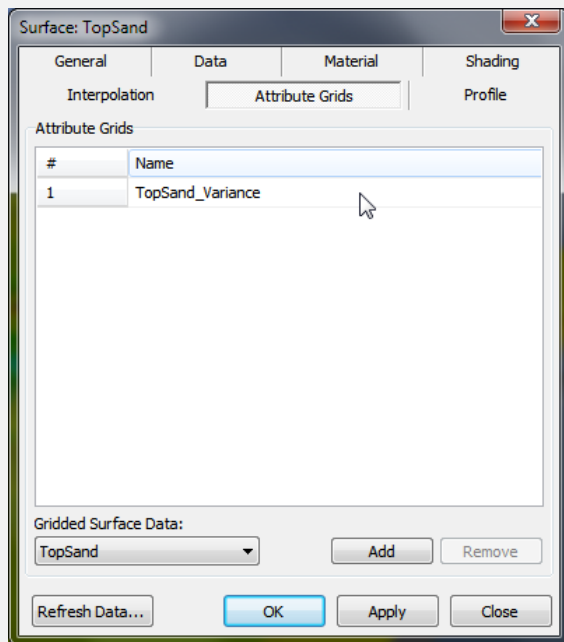
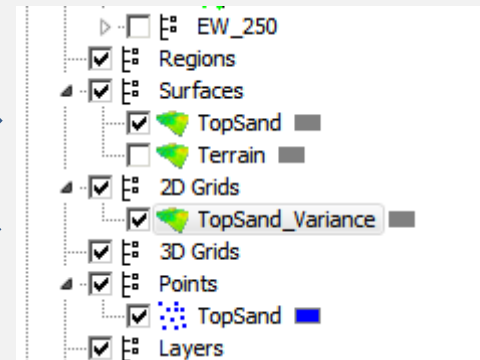


Variance Grid:

Check to produce a Variance Grid for the result.

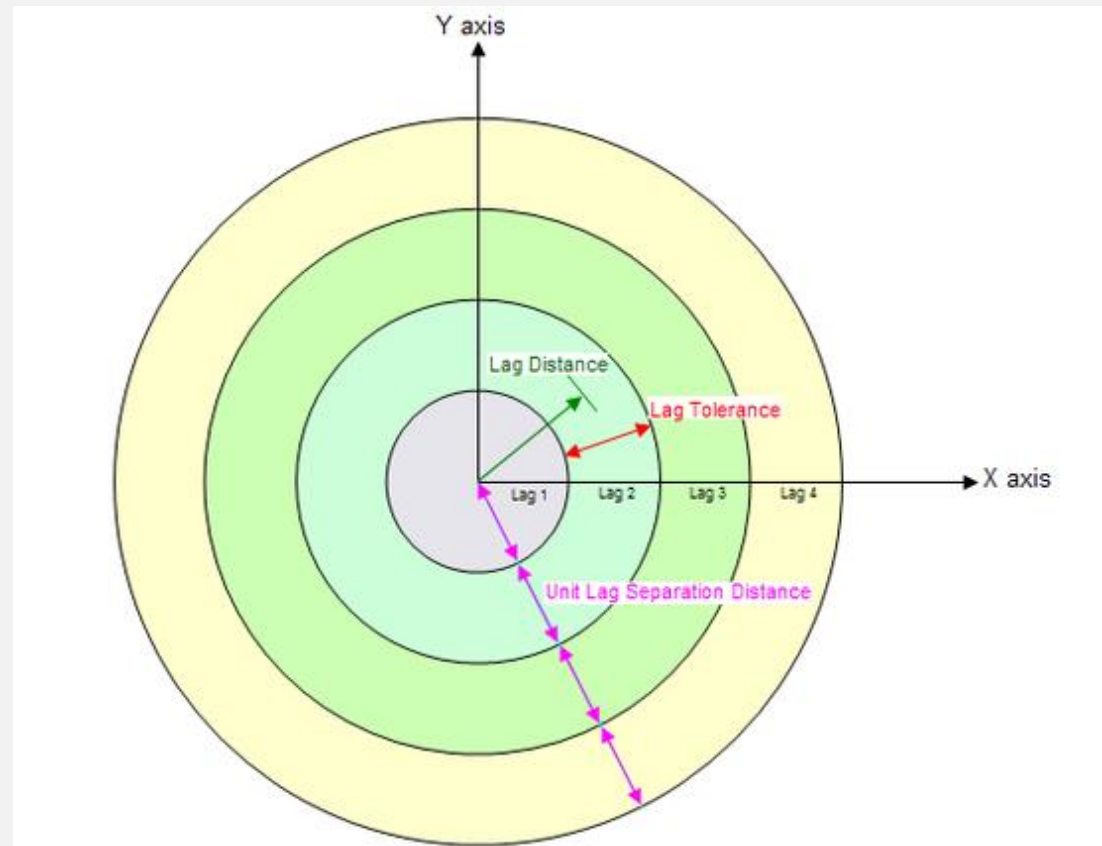
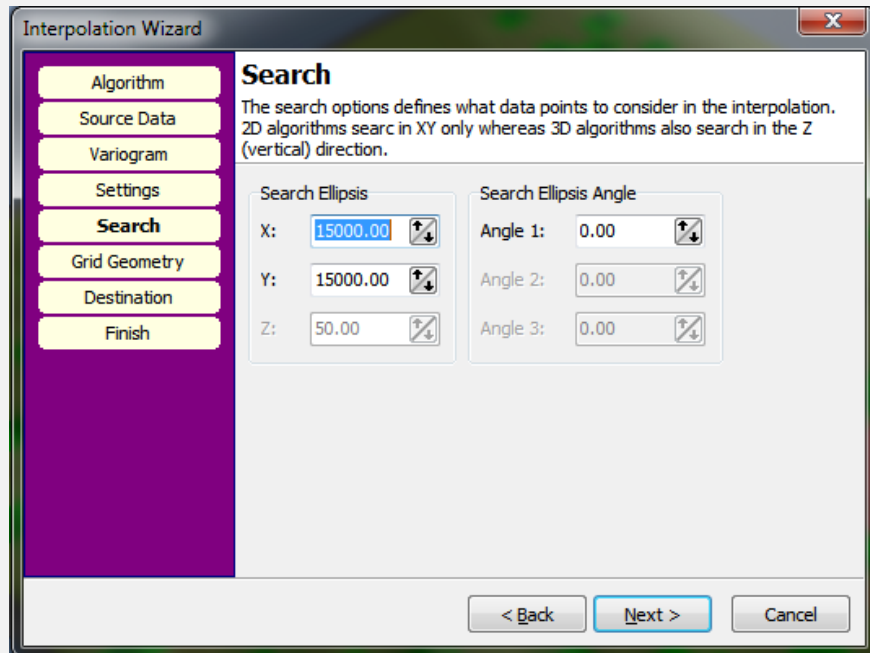
Surface →

Variance →



- Use it to check result
- Distribution of points
- Uncertainty analysis

Search



Define the Search Ellipses

- Remember distribution of data in Scatter plot !!
- Use "Data Count Per Search" to restrict to many points
- If using an Angle – experiment first with the experimental variogram to check for anisotropy !

Search

Interpolation Wizard

Grid Geometry

Specify the size and geometry of the resulting output grid. 2D grids must have identical node spacing in XY.

Grid Extent (corner node coordinates)

X Min: 1055400.00 X Max: 1062300.00 Adjust

Y Min: 1954300.00 Y Max: 1959800.00 Data Extent

Z Min: 0.00 Z Max: 5.00 Scene Extent

Node Spacing

X: 100.00 Y: 100.00 Z: 5.00

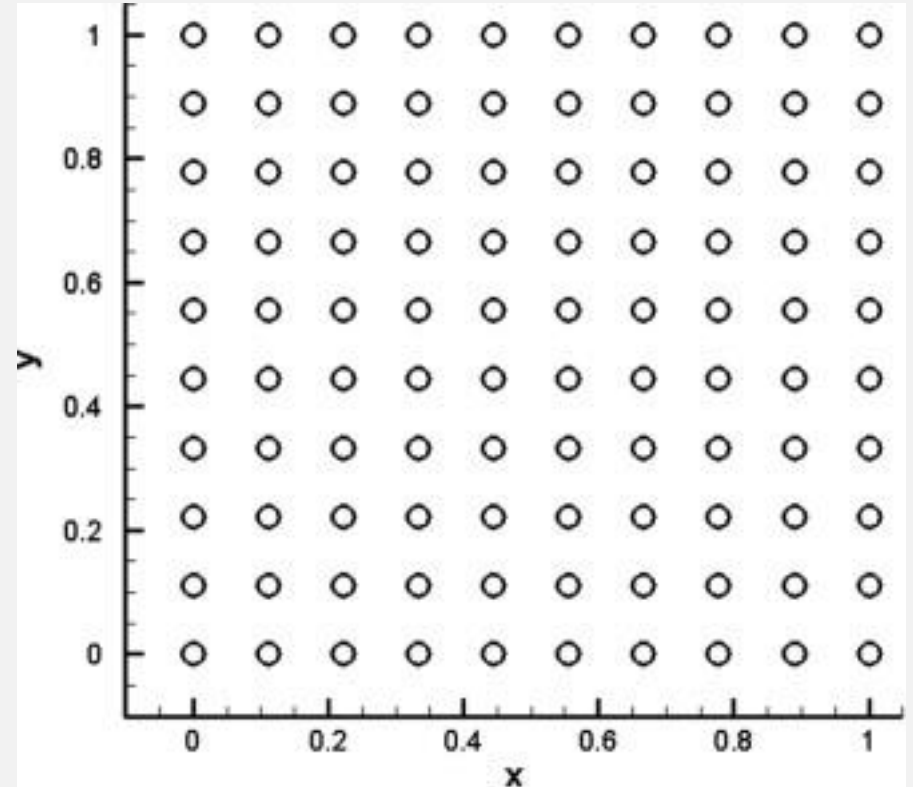
Node Count

X: 70 Y: 56 Z: 2

Grid Info

Width (X): 6900.00
Height (Y): 5500.00
Depth (Z): 5.00
Nodes: 3920

< Back Next > Cancel



Define Resulting Grid

- Again - remember distribution of data in Scatter plot !!
- How detailed features do you wish (or are able) to resolve ?
- WARNING: Small grid cells potentially leads to very large grids !
- Be conservative!

Interpolation Wizard

Destination

The interpolation creates a grid file that is saved to disk. This file is then automatically added to the scene using the dataset name and object name specified here.

☒ New Surface Object

Grid Filename(*.grd):
D:\Thailand\Interpolation\Grids\TopSand.grd

Surface Object Name: TopSand Grid Dataset Name: TopSand_Grid

Object Group Node (optional):
Surfaces

☐ Replace Existing Surface Object

TopSand

< Back Next > Cancel

General Rules of Thumb

- Check the data distribution – is it Gaussian?
- Check the spatial distribution of data
- Use the simplest model to fit the variogram
- If “islands” appears in you results, check the range used in the modeling !!
- Check that SILL corresponds to the variance you are expecting

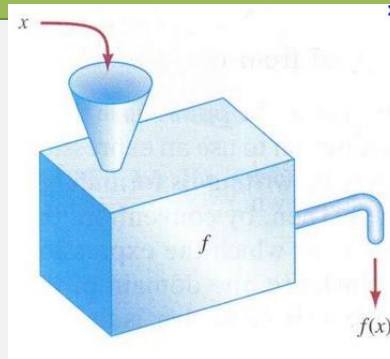
Be aware that your choices in building the variogram deeply impacts the final result
Many variables to consider – **so be careful !**

Kriging is a form of ART!



Agenda

- Interpolation defined
- General workflow in GeoScene3D
- Interpolation methods: Nearest-Neighbour (NN)
- Interpolation methods: Inverse Distance Weighting
- Interpolation methods: Kriging
- Summing up



We have our data

We have our empty grid

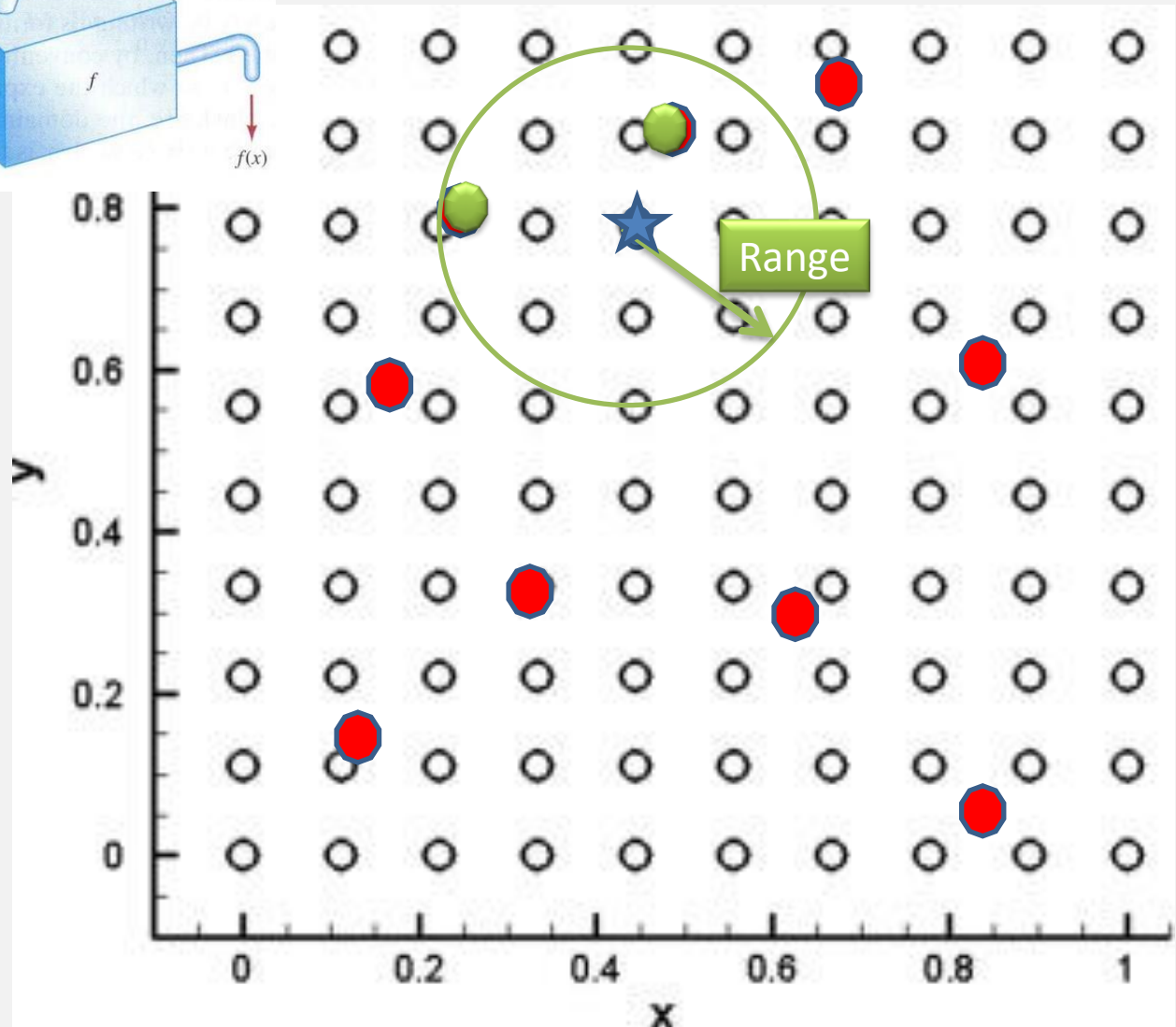
We estimate values in each node

Using a interpolation method

Which have a search range

**Which we use to find the data
to use in the interpolation.**

**And calculate an estimate
In the cell node!**



NN

- Simple - assign nearest value

IDW

- Provides smooth surfaces - assigns a value based on distance weight
- Removes trends in data
- Beware of range issues - can be handled using smoothing factor
- High Power=High Frequency; Low Power=Low Frequency

Kriging

- Statistical method - gives a value AND a measure of uncertainty
- Possibility for conserving trends in data (anisotropy)
- Variogram - a statistical tool for examining the data